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VARIATION AND INHERITANCE IN THE PARTHENOGENETIC GENERATIONS OF THE APHIS "HYALOPTERUS TRIRHODUS" (WALKER).

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(1) *Material and Methods.*

Two years ago I made a preliminary investigation* on variation and inheritance among the parthenogenetic generations of *Daphnia magna*. In conformity with Weismann's views the individuals of the same parthenogenetic family ought to resemble one another very closely indeed, but this was found to be far from the case. The variability of the members of such a family was not very strikingly smaller than among sexually produced offspring. The intensity of inheritance, however, between offspring and parthenogenetic mother appeared very considerable; and with the data that were then available it seemed to be greater than with sexually produced young.

The object of the present investigation was to test the conclusions which were drawn from the daphnia results, but the supposed high value of the parental correlation had lost much of its apparent significance by a wider acquaintance with inheritance among sexually produced offspring.

Last July the great abundance of green-fly on some common columbines in a garden suggested that the *Aquilegia* might prove to be a suitable plant for breeding purposes. A dozen large plants in pots were obtained from a florist and placed on a table under an awning in a sheltered corner of a garden. One or two of these plants had plenty of *Hyalopterus*† on them while the remainder were very nearly clean. I placed a few aphides here and there over the clean plants and these individuals soon settled down and became quiescent.

* *R. S. Proc.* Vol. 65, 1899.

† *Monograph of the British Aphides*, G. B. Buckton, Vol. II. p. 114.

The weather at that time was very warm and dry but the air around the plants was kept moist by constantly drenching the table and the surrounding ground. On watching from day to day the widely scattered and perfectly quiescent individuals one could see the young being born, and when a little cluster of 6—12 young had been produced by a mature female the mother was carefully taken off and preserved in absolute alcohol. The brood on the leaf was then enclosed in a little cage of bolting-cloth which was firmly tied around the petiole. On previous occasions I have used cylinders of very thin glass with muslin tied over the top and bottom: in these cages, however, there was so much condensation of water that the animals were often drowned. Bolting-cloth cages answered exceedingly well, they stand away from the leaf, the observer can see through them, and there is no injurious condensation.

After about a dozen families had been caged in the manner described the plants had all become so infested that it was not safe to assume that any given cluster of young had all been produced by the adjacent mother. After this, a mature female was placed on a carefully cleaned leaf and enclosed in a cage. When the individual had produced a batch of young it was removed and preserved in alcohol and the brood was allowed to grow up. I believe the cultures were absolutely pure, for the number of young enclosed in the cages was always counted and registered, and afterwards compared with the number in the adult broods. When the brood had become mature, grandchildren of the original parent had been produced, and the adult individuals were preserved. Often by this time the leaf had become somewhat faded, and when such was the case the leaf with the grandchildren was cut off and placed on a fresh leaf and the whole enclosed in a cage. Frequently under such circumstances the whole brood died, for the young did not always seem capable of migrating to the adjacent leaf. Out of 60 broods I only obtained 30 sets of grandchildren.

The Measurements.

The dimensions selected for measurement were: (1) the distance between the eyes *AB* (Fig. 1) and (2) the length of the right antenna *CD*. Body-lengths such as *XY* or *AZ* were unreliable owing to the softness and consequent contractility of the hinder region of the body; sometimes the segments comprised in *ZY* were more or less telescoped inwards.

The specimen in a drop of spirit was placed in a well-slide and covered with a small piece of thin cover-slip and the dimensions were read off by an ocular micrometer under an A Zeiss objective. The magnification was such that the measurements were made to the accuracy of ·00221 of a millimetre*. Fortunately the shape of the animal is such that there was a strong tendency for

* Subsequently for the purpose of calculating the constants it was found that the variability was so considerable that the unit employed was ·0221 or ·0442 mm.

the line AB to lie horizontally. When AB and CD were not horizontal slight adjustment of the cover-slip would almost always render them so and this was effected not through pressure exerted on the body but by alteration in the position of the body as a whole, for the legs of the animal projecting out in various

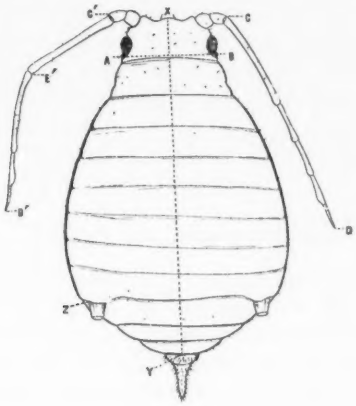

FIG. 1.

directions assured the necessary contact of the object both with the slide and the cover-slip. The antennae were as a rule very fairly straight; when curved the bend was generally at one of the joints so that the measurement could be made by adding together two lengths (see Fig. 1, $C'E' + E'D'$).

The absolute measurements AB (*Frontal Breadth*) and CD (*Length of R. Antenna*) and the ratio $\frac{CD}{AB}$ were all three dealt with in calculating the variability and the intensity of inheritance.

(2) *Growth.*

As the aphis grows the antennae become relatively longer, in other words the ratio $\frac{CD}{AB}$ rises. On this account it was essential, both for the ratio and the absolute dimensions, that all the parents and offspring should be measured as nearly as possible at the same stage of growth. In describing above how the experiment was conducted I used the words "mature female." Now, growth by no means ceases on the first production of young and I will define a "mature female" as one in which growth has become exceedingly slow. Growth at one period of life is very rapid, then it becomes slower and slower, but probably among many invertebrates it never entirely ceases during the whole life of the animal. A mature female then is one which has arrived at that stage of life at which the period of

rapid growth has passed. Among *immature* females with a frontal breadth of 12—13 units the length of the antenna tended to be about five-thirds of the frontal breadth; among *mature* females, that had practically ceased to grow, with a frontal breadth of 16—17 units, the length of the antenna had risen to about seven-thirds of the frontal breadth. To insure that the individuals preserved were mature I generally left the families until one or more of the members began to die of old age. To emphasize the importance of this procedure it may be added that since the value of the ratio rises with the growth of the mother there could easily be introduced into the correlation between offspring and mothers a wholly spurious correlation, which has no reference to heredity, by inserting into the series a group of immature mothers with offspring at the same immature stage.

(3) *Influence of Environment.*

During the period when the mothers were selected the environment was very favourable, the temperature was high and the foliage of the plants was in good condition. Later, while the offspring were growing and especially when the grandchildren were developing the conditions were much less favourable. Much rain and a considerable fall of temperature occurred, also the foliage of the columbines was withering, as is always the case in late summer.

The unfavourable conditions of the environment progressively increasing had a marked effect on the absolute size attained by the *mature* individuals; this is well seen by comparing the third and fifth columns with the second in the accompanying table. In the second generation the absolute length of the antenna was not diminished in the same proportion as the breadth, and hence the mean of the ratios (23·469) was higher than in the parents. In the third generation we see the great effect of the environment, the means of the absolute dimensions of the *mature* individuals being very conspicuously lower than in the first and second generations.

TABLE I.

Means.

Dimension	Mothers (60) weighted with their fertility	Children (368)	Grandmothers (30) weighted with their fertility	Grandchildren (291)
Frontal Breadth (*AB*) Unit = ·01 mm.	37·646	35·898	37·562	33·932
Length of R. Antenna (*CD*) Unit = ·01 mm.	85·263	84·336	83·907	76·586
Ratio $\frac{CD}{AB} \times 10$	22·670	23·469	22·464	22·567

The normal duration of life of my aphides was about three weeks to a month. If during the period of active growth the environment is unfavourable the individuals become permanently small and will never attain to a considerable size. They apparently become mature and will live three weeks or more, but will ultimately die undersized however favourable the environment may afterwards become.

On account of this great sensitiveness of the organism to its environment it can be seen that to measure the correct correlation between offspring and parent (at least with respect to absolute dimensions) all the broods of the generation ought to experience an identical environment. The fact that the conditions of life were progressively less favourable, more especially with the grandchildren, would not necessarily alter the correlation if all the broods were affected alike. In practice it is not wholly possible to provide an identical environment for all the broods; temperature, humidity of the air, illumination, etc., were nearly the same for all the broods, but the leaves of a plant differ from one another very considerably in succulence, the amount of decomposition of the chlorophyll, etc. To eliminate as much as possible these differences I always endeavoured to select similar leaves for the broods to live on.

The differences which were observed among the individuals of the generation, when growth had practically ceased, must be ascribed to the laws of variation and inheritance, and it is to these differences that I shall return when discussing the variability and the intensity of inheritance in parthenogenetically produced offspring.

(4) *Fertility.*

The birth of young by a mature female is not a continuous process, a brood of about 6—12 offspring is produced in two or three days and then there is a pause before others are born. It is probable that the number of individuals in a brood

TABLE II.

No. in Brood	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	No. of Families	Total No. of Offspring
Frequency of newly born	—	2	8	15	7	7	9	2	6	1	2	1	60	522
Frequency of adults ...	3	7	8	19	7	5	5	1	2	1	2	—	60	455
No. of winged females ...	—	—	—	2	3	—	1	—	—	—	—	3	9 in all	

bears a definite relation to the fertility of the mother which produced it. In the second row of Table II. is given the frequency of the number of individuals in the newly born families and in the third row the frequency of the adult broods. If

the frequency of the newly born broods be exhibited in a graphical form (Fig. 2) we see that the polygon conforms to the usual type of a fertility curve—there is marked skewness, the mean being considerably larger than the modal value.

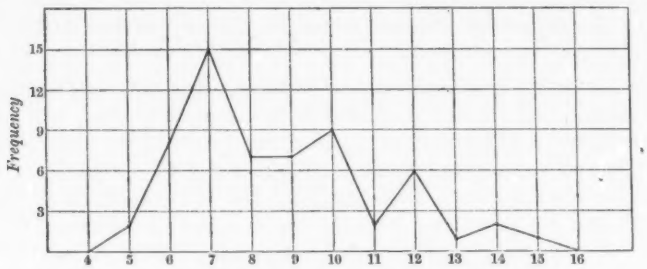

Fig. 2. Fertility Polygon. 60 Mothers, 522 Offspring.

In order to discover whether fertility was in any way related to the dimensions measured, correlation tables were prepared for the dimensions of the mother and the number in the brood (Table II A. Nos. 1, 2, 3). The coefficients of correlation are seen to be small, about $\cdot 12 \pm \cdot 09$, but since the probable errors are so great we can only assert that the larger individuals appeared to be slightly more fertile than the smaller ones.

(5) *Death-Rate.*

The total number of offspring registered was 522. Out of this number 455 individuals grew up and were collected. In the population of 455 there were nine winged individuals and 78 large corpses. The winged females appeared in only four families. The large corpses were the bodies of individuals which had doubtless died of old age, for I purposely allowed the broods to live as long as possible so as to insure that the individuals collected were as mature as they would become. The remaining 368 animals were measured.

Those that died before growing up, 67 in all, probably succumbed from weakness in constitution, etc. This gives a death-rate of 12·8 per cent. for the second generation.

To ascertain whether there was any relation between the death-rate of the brood and the deviation of the mother the percentage death-rate for every family was calculated and correlation tables for this percentage and the dimensions of the parents were prepared (Table II A. Nos. 4, 5, 6). The coefficients of correlation for the absolute dimensions are about $-\cdot 20 \pm \cdot 084$, that is the larger mothers tended to have healthier offspring.

It was also desirable to know whether deviation in adult families was associated with increased death-rate during their growth. The mean ratio of each family

was calculated and a correlation table was made between it and the percentage death-rate, the coefficient is so small (no. 7) and the probable error so great that we can only say that perhaps there was a slight tendency for families with high ratio to have a small death-rate. We may assume that adult families which exhibited considerable divergence were parts of families, which if all the members had survived would have shown the same or greater divergence, hence we are justified in the assumption that the deaths which occurred were not at all, or very slightly selective with respect to the dimensions with which we are here concerned.

TABLE II A.

Number	x	y	r_{xy}
1	Length of Antenna of Mother	Number in newly born Brood ...	·120 ± ·086
2	Frontal Breadth of Mother ...	Number in adult Brood	·129 ± ·086
3	Ratio of Mother	" " "	·119 ± ·086
4	Length of Antenna of Mother	Percentage Death-Rate of Family	− ·201 ± ·084
5	Frontal Breadth of Mother ...	" " " "	− ·184 ± ·084
6	Ratio of Mother	" " " "	− ·070 ± ·087
7	Mean ratio of Family ...	" " " "	− ·113 ± ·086
8	Number in newly born Brood	" " " "	− ·188 ± ·084
9	Difference between the ratio of Mother and mean ratio of Family	" " " "	− ·128 ± ·086
10	Difference between the ratio of Mother and mean ratio of Family	Number in newly born Brood ...	− ·007 ± ·087

Further, the fertility of the mother, measured by the number in the newly born brood, was apparently slightly correlated with the death-rate (no. 8), that is offspring of fertile mothers tended to be healthier than those of less fertile mothers.

Also, families in which the mean ratio diverged widely from the ratio of the parent tended to be healthier (no. 9) than those families which more nearly resembled the parent; but there was apparently no correlation between this resemblance and the fertility of the mother (no. 10).

(6) *Organic Correlations.*

For any consideration of the correlation of one organ of a parent with some other organ of the offspring it would be interesting to ascertain the inter-relations of the parts in the individual. The correlation table for frontal breadth (AB) and length of antenna (CD) of the second generation is given in Table III. The standard deviation of $AB = 1{\cdot}3415$ and of $CD = 2{\cdot}4744$; the coefficient of correlation is ·8023 ± ·0125. Hence the two dimensions are closely correlated.

TABLE III.

Frontal Breadth (AB) and Length of Antenna (CD), Second Generation.

Length of R. Antenna: Unit = 2 divisions, = ·0442 mm.

Frontal Breadth: Unit = 1 division of Ocular Micrometer = ·0221 mm.	24·0–25·9	26·0–27·9	28·0–29·9	30·0–31·9	32·0–33·9	34·0–35·9	36·0–37·9	38·0–39·9	40·0–41·9	42·0–43·9	44·0–45·9	46·0–47·9	48·0–49·9	50·0–51·9	Totals
12·0—12·9	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
13·0—13·9	1	0	4	4	2	2	—	—	—	—	—	—	—	—	13
14·0—14·9	—	2	7	13	9	9	7	1	—	—	—	—	—	—	48
15·0—15·9	—	—	1	12	20	24	18	17	5	—	—	—	—	—	97
16·0—16·9	—	—	—	1	2	7	22	25	22	7	2	2	—	—	90
17·0—17·9	—	—	—	—	—	—	8	13	16	23	17	6	2	—	85
18·0—18·9	—	—	—	—	—	—	—	—	7	3	9	4	3	1	27
19·0—19·9	—	—	—	—	—	—	—	—	1	0	3	3	—	—	7
Totals	1	2	12	31	33	42	55	56	51	33	31	15	5	1	368

The smaller individuals in the series, which, however, appeared mature in that they had very nearly ceased to grow, tended to have an antenna relatively shorter than in the larger animals. I took the frontal breadth as a criterion of the general size of the body and instituted a correlation table between it (AB) and the ratio $\left(\frac{CD}{AB}\right)$, Table IV. Since AB is the same number in both cases there should be a negative correlation if there be no organic correlation. As a matter of fact the coefficient of correlation is $+ \cdot 3200 \pm \cdot 0316$; the standard deviation of $AB = 1 \cdot 3415$ and of $\frac{CD}{AB} = 1 \cdot 8363$.

TABLE IV.

Frontal Breadth (AB) and Ratio $\left(\frac{CD}{AB}\right)$, *Second Generation.*

Ratio: Unit = 1 hundredth of Breadth.

Frontal Breadth: Unit = 1 division of Ocular Micrometer = ·0221 mm.	18·0–18·9	19·0–19·9	20·0–20·9	21·0–21·9	22·0–22·9	23·0–23·9	24·0–24·9	25·0–25·9	26·0–26·9	27·0–27·9	28·0–28·9	Totals
12·0—12·9	—	—	—	—	—	—	1	—	—	—	—	1
13·0—13·9	1	0	0	5	3	0	3	0	1	—	—	13
14·0—14·9	—	4	8	8	8	6	8	4	1	1	—	48
15·0—15·9	—	6	8	19	16	19	18	7	4	—	—	97
16·0—16·9	—	1	2	7	19	23	18	14	3	1	2	90
17·0—17·9	—	—	—	8	13	16	15	23	8	1	1	85
18·0—18·9	—	—	—	2	4	4	8	5	3	0	1	27
19·0—19·9	—	—	—	1	0	2	4	—	—	—	—	7
Totals	1	11	18	50	63	70	75	53	20	3	4	368

Professor Pearson* has introduced a formula for the purpose of extracting this "spurious correlation." If $\frac{x_1}{x_3}$, $\frac{x_2}{x_4}$ are two indices; v_1, v_2, v_3, v_4 coefficients of variation $\left(\frac{\text{standard deviation}}{\text{mean}} \times 100\right)$ of x_1, x_2, x_3, x_4 respectively; r_{12}, r_{23}, r_{34} coefficients of correlation between x_1 and x_2, x_2 and x_3, x_3 and x_4 respectively and ρ the coefficient of correlation between the indices, then

$$\rho = \frac{r_{12}v_1v_2 - r_{14}v_1v_4 - r_{23}v_2v_3 + r_{34}v_3v_4}{\sqrt{v_1^2 + v_3^2 - 2r_{13}v_1v_3}\sqrt{v_2^2 + v_4^2 - 2r_{24}v_2v_4}}.$$

Applying this to the present case

$$\rho = \frac{r_{AB,\,CD}\,v_{AB}\,v_{CD} - v_{AB}\,v_{AB}}{v_{AB}\sqrt{v^2_{CD} + v^2_{AB} - 2r_{CD,\,AB}\,v_{CD}\,v_{AB}}}.$$

Now, if the correlation ($r_{CD,\,AB}$) be supposed to disappear the expression nevertheless does not vanish but becomes

$$= -\frac{v_{AB}}{\sqrt{v^2_{CD} + v^2_{AB}}} = \rho_0,$$

$$v_{AB} = 8{\cdot}2587, \qquad v_{CD} = 12{\cdot}9682,$$

$$\therefore \rho_0 = -{\cdot}5372,$$

and

$$\rho - \rho_0 = {\cdot}3200 + {\cdot}5372 = +{\cdot}8572.$$

Assuming that the expression $\rho - \rho_0$ has a definite meaning, we conclude that there is a strong correlation between the ratio and the size of the body in the adult animal.

(7) *The Variability of the Race.*

The coefficient of variation of any organ has been defined by Professor Pearson† as $\frac{\text{standard deviation}}{\text{mean}} \times 100$; it is thus a ratio-measure of variability while the standard deviation is an absolute measure of the same. In the accompanying table the standard deviations and coefficients of variation are given for parents and offspring.

The variability of the second generation (fourth and fifth columns, Table V.) is greater than that of the parents, and this we should expect since certain selective influences would doubtless be less active than among wild aphides. The fact that the mothers are weighted with their fertility will have no appreciable effect on the standard deviation, since as was mentioned above fertility is only slightly correlated with any of the dimensions.

* *Proc. Roy. Soc.* 1897.

† *Phil. Trans. of the Roy. Society*, Vol. 187 (1896) A, p. 277.

TABLE V.

Dimension	Mothers (60) weighted with their fertility		Children (368)		Grandmothers (30) weighted with their fertility		Grandchildren (291)	
	S. D.	Coefficient of Variation	S. D.	Coefficient of Variation	S. D.	Coefficient of Variation	S. D.	Coefficient of Variation
Frontal Breadth (AB) Unit = ·01 mm.	2·7682	7·3533	2·9647	8·2587	2·2815	6·0739	1·9099	5·6283
Length of R. Antenna Unit = ·01 mm.	9·2262	10·8205	10·9367	12·9682	7·3593	8·7707	7·9972	10·4422
Ratio $\frac{CD}{AB} \times 10$	1·4974	6·6054	1·8363	7·8244	1·2735	5·6692	1·6309	7·2270

We must compare the variability of the third generation (eighth and ninth columns) with the variability of the grandparents (sixth and seventh columns), the fewness of which render the values somewhat aberrant. The diminished variability of the frontal breadth (5·6283) of the third generation must be referred to the direct action of the environment, for we have already seen that the external conditions have an important influence on the ultimate size attained by the mature offspring, and apparently the standard deviation is also reduced in the case of our criterion of size, the frontal breadth.

It is interesting to notice that here as in man* the variability of children is greater than that of parents, the adul· children being a population only partially selected. Similarly the variability of the grandchildren is greater than that of their grandparents (with the exception of the frontal breadth mentioned above). Also the variability of the grandmothers is considerably less than that of the mothers, for the grandmothers are a doubly selected population, being in fact only those individuals which are capable of producing *fertile* offspring.

These results prove that the variability of the race, measured by the coefficient of variation, is by no means smaller than among sexually reproduced forms. This coefficient varies considerably for different characters; to mention a few examples the following may be cited. Prof. Pearson and others have shown that for stature in man† the coefficient of variation varies from about 3—4, for the length of the long bones‡ from 4—6. Breadth of wing of 98 wild ♂ *Strenia clathrata*§ had a coefficient of 4·57.

* *Phil. Trans. Roy. Soc.* Vol. 187 (1896) A, p. 270.

† *Ibid.* p. 278.

‡ Lee and Pearson, *R. S. Proc.* 1897, Vol. 61, p. 348. Warren, *Phil. Trans. of the Roy. Soc.* 1898, Vol. 189, B, p. 140.

§ Not yet published.

(8) *Inheritance. Correlation between Parent and Offspring.*

The question now to be discussed is how far peculiarities in the parent are transmitted to the offspring and succeeding generation in parthenogenesis. In Tables VI. to XI. (placed at the end of the present paper) are given the correlation tables for the absolute measurements and the ratio.

The coefficients of *correlation* (r) were calculated from the formula $r = \frac{\Sigma(xy)}{n\sigma_1\sigma_2}$, where σ_1 and σ_2 are the standard deviations of parent and offspring respectively; and the coefficients of *regression* of offspring on mothers $= r\frac{\sigma_2}{\sigma_1}$. In the accompanying table the various results are tabulated together with their probable errors.

TABLE XII.

Genus	Dimension	Parents and Offspring (368)		Grandparents and Grandchildren (291)	
		Coefficient of Correlation	Coefficient of Regression	Coefficient of Correlation	Coefficient of Regression
Hyalopterus	Frontal Breadth (AB) ...	·3354 ± ·0312	·3592 ± ·0355	·3208 ± ·0355	·2685 ± ·0314
	Length of R. Antenna (CD)	·4273 ± ·0287	·5065 ± ·0377	·1766 ± ·0383	·1919 ± ·0244
	Ratio $\left(\frac{CD}{AB}\right)$	·4392 ± ·0284	·5385 ± ·0387	·2305 ± ·0374	·2952 ± ·0493
Daphnia	Ratio $\frac{\text{Length of Protopodite}}{\text{Length of body}}$	·466* ± ·054	·619 ± ·0809	[·27 ± ·12†	·5 ± ·2]
Mean of Coefficients		·41	·50	·24	·25

To this table are also added the results obtained from Daphnia. The means of the correlation between parents and the second and third generations are respectively ·41 and ·24; the grandparental correlation for Daphnia has been omitted, since the probable error is so very large.

Such correlation, and especially that of the grandparental, is distinctly too high for Galton's Law for sexual inheritance in its *simple* form, and the question arises whether inheritance is stronger in parthenogenetic than in sexual reproduc-

* 96 young, 23 mothers. † 26 young, 7 grandparents.

tion. But a wide survey of the correlation of very various characters between parent and offspring of sexual forms lends no support to such a view. It should be noticed that the parental correlation for the ratio approximates more closely to the value usually found among sexual forms (·45—·50)* than do the coefficients of either of the absolute dimensions. It is very probable that these discrepancies must be referred to the effect of environment. This view is favoured by the following consideration. I selected the frontal breadth as a criterion of the general size of the body and it is the size of the animal that is most affected by the environment. Now, the coefficients for the frontal breadth are more aberrant than those for the length of the antenna, and so it is reasonable to assume that these deviations are to be ascribed to the varying conditions of life.

Figs. 3, 4 and 5 represent graphically the correspondence between the line of means of the arrays and the correlation line. Theoretically the mean of each array near the region of the general mean of the parents should be on the dotted line. Further away from the parental mean the means of the arrays are much less likely to approximate to the dotted line for the following reasons: (1) the unavoidable fewness of the more diverging individuals and the consequent impossibility of obtaining a satisfactory mean to these arrays, and (2) in the case of skew curves the correlation line should really be sigmoid and not straight.

In Fig. 3 the divergence of the uppermost point from the dotted line must be ascribed to the small number of individuals, there being only 7 members of this array (Table X.). In Fig. 4 the widely deviating point between divisions + 2 and + 3 (array 42·0—43·9, Table VIII.) is probably due to the circumstance that these families may have lived on particularly innutritious leaves, and this view is favoured by the fact that the adult individuals of these families tended to be small.

(9) *Cross-Inheritance.*

Although two organs A and B of an animal may be correlated yet it does not necessarily follow that there is a correlation between the organ A of the parent and B of the offspring. If, however, A and B are closely correlated in the individual it is probable that there will generally be found some cross-inheritance between A or B of the parent and B or A of the offspring.

If A (frontal breadth) of the offspring be directly acted upon by the environment and A and B (length of antenna) are dimensions closely correlated in the individual, then the irregular alteration of A in the offspring by different environments for different broods would probably affect the correlation between organ B of the parent and the same organ of the offspring. If the alteration be uniform, that is if all the offspring are acted upon by one and the same environment which

* Pearson, *R. S. Proc.* 1900, p. 157. Pearson and Lee, *Phil. Trans. of the Roy. Soc.*, Vol. 195 (A), 1900, p. 119.

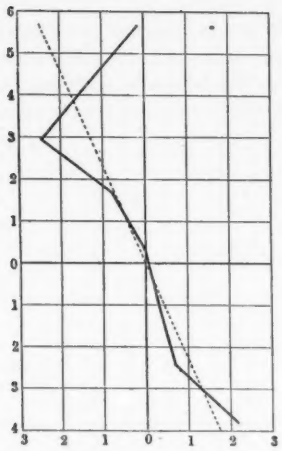

FIG. 3. Parents and Offspring. Ratio. 1 division $= \frac{1}{2}$ standard deviation.

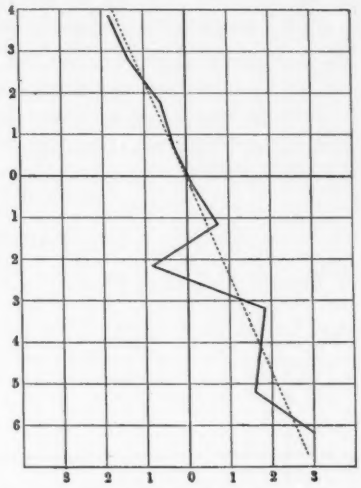

FIG. 4. Parents and Offspring. Absolute length of Antenna. 1 division $= \frac{1}{2}$ standard deviation.

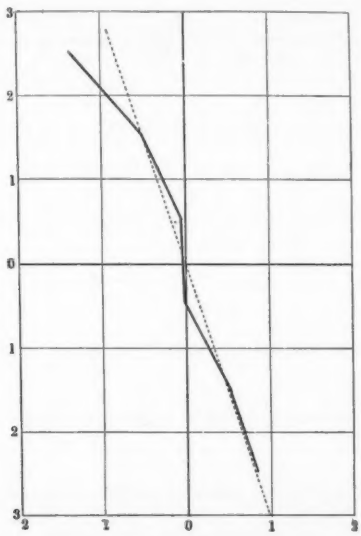

FIG. 5. Frontal Breadth : Grandparents and Grandchildren. 1 division = the standard deviation.

is different from that to which the parents were subjected, then the correlation of organ *B* between parent and offspring would not necessarily be changed.

We saw above that the correlation between frontal breadth and length of antenna *in the individual* was ·8023, and now we wish to know the correlation between the frontal breadth of the mother and the length of antenna of the offspring. The correlation table is given in Table XIII.; the coefficient of correlation is ·3004 ± ·0320.

TABLE XIII.

Frontal Breadth of Parents and Length of Antenna of Offspring.

Offspring: Length of Antenna. Unit = 2 divisions of Ocular Micrometer = ·0442 mm.

Parents: Frontal Breadth. Unit = 1 division of Ocular Micrometer = ·0221 mm.	24·0–25·9	26·0–27·9	28·0–29·9	30·0–31·9	32·0–33·9	34·0–35·9	36·0–37·9	38·0–39·9	40·0–41·9	42·0–43·9	44·0–45·9	46·0–47·9	48·0–49·9	50·0–51·9	Weighted Mothers	Number of Families
14·0—14·9	—	—	2	3	1	—	—	—	—	—	—	—	—	—	6	1
15·0—15·9	—	2	4	7	6	8	6	8	8	2	3	5	1	1	61	9
16·0—16·9	1	0	5	9	13	15	28	27	20	9	7	2	2	—	138	25
17·0—17·9	—	—	1	9	7	8	8	9	9	8	8	1	—	—	68	11
18·0—18·9	—	—	—	3	6	11	13	11	13	10	5	2	1	—	75	11
19·0—19·9	—	—	—	—	—	—	—	1	1	2	3	3	—	—	10	2
20·0—20·9	—	—	—	—	—	—	—	—	—	2	5	2	1	—	10	1
Totals	1	2	12	31	33	42	55	56	51	33	31	15	5	1	368	60

Also we saw that the correlation between the frontal breadth (*AB*) and the ratio $\frac{CD}{AB}$ of the individual was + ·3200. In Table XIV. is shown the correlation distribution for the frontal breadth of mother and the ratio of offspring; the coefficient of correlation is ·1688 ± ·0342.

Now let us calculate the values of the cross-correlations by aid of Professor Pearson's theorem* that:

Cross-inheritance = mean of direct inheritances × organic correlation.

We find in the first case:

$$\text{Cross-correlation} = \tfrac{1}{2}(\cdot3354 + \cdot4273) \times \cdot8023 = \cdot3059,$$

and in the second case:

$$\text{Cross-correlation} = \tfrac{1}{2}(\cdot3354 + \cdot4392) \times \cdot3200 = \cdot1239.$$

The first result is in good agreement and the second as close as could be anticipated from the large probable errors of our results.

* *R. S. Proc.* Vol. 62, p. 411.

TABLE XIV.

Frontal Breadth of Parents and Ratio of Offspring.

Offspring: Ratio. Unit = 10 thousandths of Frontal Breadth.

Parents: Frontal Breadth. Unit = 1 division of Ocular Micrometer = ·0221 mm.	18·0–18·9	19·0–19·9	20·0–20·9	21·0–21·9	22·0–22·9	23·0–23·9	24·0–24·9	25·0–25·9	26·0–26·9	27·0–27·9	28·0–28·9	Weighted Mothers	Number of Families
14·0—14·9	—	—	—	3	2	0	1	—	—	—	—	6	1
15·0—15·9	—	3	8	5	4	10	13	7	7	0	4	61	9
16·0—16·9	1	5	6	25	27	33	23	13	4	1	—	138	25
17·0—17·9	—	2	2	10	18	8	12	11	4	1	—	68	11
18·0—18·9	—	1	2	7	12	18	17	14	4	—	—	75	11
19·0—19·9	—	—	—	—	—	1	2	6	1	—	—	10	2
20·0—20·9	—	—	—	—	—	—	7	2	0	1	—	10	1
Totals	1	11	18	50	63	70	75	53	20	3	4	368	60

(10) *The Variability among the Members of a Brood.*

We have already seen that the variability of the race is in no way smaller than in sexual animals. It is now necessary to discuss the variability in the individual family.

The standard deviation for all the families containing 7 or more members was calculated and the mean is given in the last column but one of the accompanying table (Table XV.).

TABLE XV.

Genus	Dimension	Variability	Parents (60) weighted with their fertility	Offspring (368)	Arrays $\sigma\sqrt{1-r^2}$	Mean of the S.D.'s of all families over and including 7	Total No. of families in which the S.D. was calculated
Hyalopterus	Breadth ... ... unit = ·01 mm.	S.D. ...	2·7682	2·9647	2·7930	1·5890	22 families and 190 individuals
		Percentage	57·4	53·6	56·9	100	
	Length of Antenna unit = ·01 mm.	S.D. ...	9·2262	10·9367	9·8884	5·4839	
		Percentage	59·4	50·1	55·5	100	
	Ratio ... ... unit = ·010 of Frontal Breadth	S.D. ...	1·4974	1·8363	1·6797	·9953	
		Percentage	66·5	54·2	59·2	100	
Daphnia	Ratio ... ... unit = ·002 of Body Length	S.D. ...	2·2208	2·9503	2·6104	1·2747	22 families, 95 individuals
		Percentage	57·4	43·2	48·8	100	
Mean of the percentages			60·2 °/₀	50·3 °/₀	55·1 °/₀	100	

In the 6th column is given the average S.D. of an array calculated from the formula $\sigma\sqrt{1-r^2}$ and in 4th and 5th columns the S.D. of parents and offspring. The general standard deviations for arrays are of considerable magnitude, being in fact greater than the S.D.'s of the parents. The mean S.D.'s of the families are much less than those of arrays. We should expect the former to be smaller; for although all the parents which produce an array are alike in the particular dimension, yet they are unlike in their ancestry.

In this table is also given the ratio (expressed as a percentage) of the mean S.D. of the families to the standard deviation of parents, offspring, and arrays. To these results are also added those obtained from daphnia; and in the bottom line of the table the means of the percentages are given.

It must be realised that the families are too small to give a very reliable determination of the standard deviation, but nevertheless the mean results probably give a rough indication of actual fact. According to these results the variability of the family is seen to be 60 °/₀ of that of all the parents, or in other words if we bred from a single individual yet more than half of the racial variability would remain.

It is unfortunate that at present there are no data available from which family variability may be deduced for sexual forms and we must await further investigations before a comparison can be made with these results from parthenogenetic families. In the meantime it may be seen that the present results are antagonistic to the views on variation and inheritance expounded by A. Sedgwick *; for with a family variability of 60 °/₀ of the racial it is difficult to see "that genetic variations cannot occur in asexual reproduction, and that if any indefinite variability recalling genetic variability makes its appearance it must be part of the genetic variability and directly traceable to the zygote from which the asexual generations started."

(11) *The Correlation between Members of the same Brood.*

Let any member of a family be chosen, then we wish to find a number which will express its probable degree of similarity to any other member taken at random from the same family. Just as the mean standard deviation of all the families would be a measure of the dissimilarity of members of a family, so the correlation coefficient for all possible pairs of brethren would be a measure of their resemblance. If the S.D. of every family be very small or 0, that is if there be very little or no variation among brethren, the correlation would be perfect and equal to 1. If, on the other hand, the S.D. is large there is much dissimilarity among brethren, and consequently the correlation would be expressed by a number much less than 1. We have already seen that, contrary to a prevailing

* Presidential Address to Section D of the British Association, 1899. Reprinted in *Nature*, Vol. 60, p. 507.

opinion, the standard deviations of families are not inconsiderable, and hence the correlation between brethren will be much less than unity.

In Tables XVI.—XXI. (at the end of the present paper) are given the correlation tables for pairs of offspring. In Table XXII. the results both for aphis and daphnia are exhibited.

TABLE XXII.

Genus	Dimension	Pairs of Offspring 368 individuals 1114 pairs	Pairs of Grandchildren 291 individuals 1894 pairs
Hyalopterus	Frontal Breadth	·6660 ± [·0112]*	·3890 ± [·0132]
	Length of Antenna	·6785 ± [·0109]	·4132 ± [·0129]
	Ratio	·5890 ± [·0132]	·3382 ± [·0137]
Daphnia	Ratio	·6934 ± [·0270]†	——
Mean of Coefficients		·656	·380

In the fourth column are given the coefficients for grandchildren. From the nature of the experiment the collections of grandchildren consisted of a mixture of *sisters* and cousins‡ in unknown proportions. The coefficients for the offspring are high, but for the following reasons further investigation is needed before it can be concluded that there is greater similarity among brethren in parthenogenesis than in sexually produced offspring. The value for the ratio is distinctly lower than for the absolute measures; as in the case of comparing the S.D. of arrays with the mean of the standard deviations of the separate families, so in the case of the correlations the values for the ratio appeared the most normal.

Each brood or collection of grandchildren lived on one leaf, and consequently every member of a family was subjected to a practically identical environment. Those that had an innutritious leaf would all tend to be small when adult and those with a particularly wholesome leaf would be large.

We have seen, however, that there is considerable correlation between the ratio and the adult size of the animal, hence even in the parental correlation of

* These probable errors have been calculated from the usual formula $\cdot 67449 \frac{1-r^2}{\sqrt{n}}$. I have made $n=1114$ but it is doubtful whether this gives an accurate probable error.

† 23 families, 168 pairs.

‡ In a *rough* sort of way the correlation of true cousins might be found from this. If r = correlation of cousins and there be m pairs of sisters and m' of cousins, then $(m+m')\ \cdot 3801 = m\ \cdot 6445 + m'r$. The proportions of sister and true cousin pairs from the total individuals and total pairs given in Table XXII, could not be widely different from the ratio 3 to 10. Hence the cousin correlation would be as high as ·3, or nearly half the fraternal correlation.

the ratio the environment might be expected to have some effect, but the greatest effect would naturally be on the fraternal correlation. Theoretically the mothers and all the broods ought to have lived on exactly the same sort of leaf, or failing this the mothers could have lived on one kind of leaf and *all* the broods on some other kind. Unfortunately this is not wholly possible in practice.

(12) *Summary.*

The present investigation well illustrates the exceeding difficulty of breeding organisms for statistical purposes, and it emphasizes the necessity for the greatest caution in drawing conclusions from the results.

The reaction of the organism to its environment is so prompt that it is practically impossible to breed two generations under such conditions as to ensure the same mean and the same variability in the two generations. The differences which occur are due to two sets of causes: (1) the unavoidable removal of certain selective agencies which would act under purely natural conditions, (2) the direct effect of the environment both on the parents and on the offspring. The resultant effect of numerous influences under either or both of these headings might increase or diminish the apparent intensity of inheritance. I will now summarize in separate paragraphs the facts and conclusions deduced from the present investigation.

(1) The mean of the ratio rises very greatly with the growth of the individual.

(2) An individual was said to be "mature" or "adult" after the period of rapid growth had passed.

(3) Under unfavourable conditions the individual grows slowly, after a time offspring may be produced, and ultimately growth nearly ceases and the individual is "mature" but permanently small.

(4) The unfavourable conditions of the environment had a marked effect on the absolute dimensions of the adult third generation, but they had little effect on the mean of the ratio. These conditions did not induce any great formation of winged individuals.

(5) The number of offspring in a brood may be taken as a rough index of the fertility of the parent. There was but little correlation between fertility and the dimensions measured on the parents.

(6) In the second generation there was a death-rate of 12·8 per cent. among the immature animals. The larger individuals tended to have healthier families. Offspring of fertile mothers were somewhat stronger in constitution than those of less fertile mothers. But those families in which the majority of the adult members were widely divergent from the filial mean exhibited little difference in their death-rate.

(7) The frontal breadth and the length of antennæ are closely correlated in the adult individual, the coefficient being ·80. The frontal breadth (taken

as a criterion of the general size) has a correlation with the ratio of +·32, but if we accept the significance of the expression $\rho - \rho_0$, there is really a much larger organic relationship than is indicated by this number.

(8) The variability of the race, measured by the coefficients of variation, was large (6—10), and there is apparently no significant difference between them and the coefficients found among sexual forms.

(9) The mean of the coefficients, for aphis and daphnia, of the parental correlation was ·41, and of the grandparental ·24; thus here again there is no marked difference between parthenogenetic and sexual reproduction.

(10) Two examples of cross-inheritance were examined: (1) frontal breadth of parent and length of antenna of offspring $r = ·30$; and (2) frontal breadth of parent and the ratio of offspring $r = ·17$. These results are in good accord with Prof. Pearson's theorem for cross-inheritance.

(11) The variability of arrays of offspring is by no means significantly small: and the variability of the individuals of a brood, measured by the mean of the standard deviations of the individual families, amounts to as much as 60 % of the racial variability. This may perhaps be rather less than in sexual reproduction, but no very satisfactory direct comparison is at present possible.

(12) A convenient measure of the similarity of brethren is afforded by the fraternal correlation. From numerous data Prof. Pearson* regards ·49 or ·50 as the average fraternal correlation among sexual forms, while the mean coefficient for aphis and daphnia is ·66; but it should be noticed that the ratio of aphis, on which the environment would have less effect, more nearly approaches the sexual value than do the absolute dimensions. The question as to whether we have here a real difference between parthenogenetic and sexual offspring can only be decided by further investigation both on aphis and other forms.

(13) *Concluding Remarks.*

The coefficient of correlation for a pair of organs must have a definite physical meaning, and if no spurious correlation be introduced, it should indicate the proportion of the manifold causes, affecting the two organs in question, which are common to each.

The causes of inheritance are probably alike in nature to the causes of the correlation of parts of an individual. Thus the right and left organs of a bilateral animal are usually strongly correlated, and this fact translated into a physical meaning must imply that the numerous causes which build up the organ on the two sides are nearly identical. Now, it seems probable that a long comparative series of observations of the correlation between different parts of the numerous organisms and the *cross-inheritance* between these various parts would throw a flood of light on the actual mechanism of both growth and inheritance.

* Pearson, *R. S. Proc.* 1900, p. 157. Pearson and Lee, *Phil. Trans. Roy. Soc.* Vol. 195 (A), p. 119.

We know from Driesch's experiments that the mere position of a cell in the blastosphere determines its fate, and that the blastomeres are at first, so to speak, merely plastic masses which are welded into an organism by the life of the structure as a whole. The failure of inheritance must be regarded as of the same nature as the failure in the growing organism to produce the normal relative proportions of the different parts of the body.

And finally, with regard to the special subject of this paper, inheritance in parthenogenesis. From the nature of the case, since there is only a female parent, the inheritance cannot be a blend of maternal and paternal characters. If it were "exclusive inheritance" without reversion the coefficients ought to be far higher than in sexual forms. But this is not the case. The inheritance could be either of the nature of a blend of ancestral characters, or the "exclusive" transmission of ancestral characters, that is reversion.

In the accompanying table in the second column I give the theoretical values deduced by Professor Pearson* for exclusive inheritance, without reversion, in the third column the values for blended inheritance or exclusive inheritance with reversion and $\gamma = 2{\cdot}35$†. In the fourth and fifth columns are the values found for eye-colour in man and coat-colour in the horse, and in the sixth column the

TABLE XXIII.

Relationship	Exclusive Inheritance, no reversion	Blended Inheritance or Exclusive Inheritance with reversion. $\gamma = 2{\cdot}35$	Man. Eye-colour	Horse. Coat-colour	Aphis and Daphnia. Mean coefficients for all dimensions	Aphis and Daphnia. Mean coefficients for ratios only
Parental ...	·50	·40	·49	·52	·41	·45
Grandparental	·25	·20	·32	·33	·24	·25
Fraternal ...	·4 to 1·0	·66	·47	·63	·66	·64

mean coefficients for aphis and daphnia. In the case of the parthenogenetic animals the parental correlation compared with the results from the sexual forms is low, and the fraternal correlation distinctly high. If we exclude the less reliable absolute dimensions and only consider the ratios, the parental correlation is raised (seventh column). The question to be decided by further investigation is whether these differences are real or are merely the effects of the reaction of the organism to its environment. So far as the present investigation goes there is no increase, rather a decrease is shown in parental inheritance as we pass from sexual to parthenogenetic forms. There is possibly an increase, but we cannot as yet assert it definitely, in fraternal correlation. On the whole the numbers for both sexual and parthenogenetic inheritance lie broadly within the same limits.

* *Phil. Trans. Roy. Soc.* Vol. 195 (A), p. 119.

† *Proc. Roy. Soc.* Vol. 66, 1900, p. 149.

TABLE VI.

Parents and Offspring.

Offspring: Frontal Breadth. Unit = 1 division of Ocular Micrometer = ·0221 mm.

Parents: Frontal Breadth. Unit = 1 division of Ocular Micrometer = ·0221 mm.	*12·0–12·9*	*13·0–13·9*	*14·0–14·9*	*15·0–15·9*	*16·0–16·9*	*17·0–17·9*	*18·0–18·9*	*19·0–19·9*	Weighted Mothers	Number of Families
14·0—14·9	—	5	1	—	—	—	—	—	6	1
15·0—15·9	—	2	18	16	14	9	2	—	61	9
16·0—16·9	—	4	13	36	46	29	7	3	138	25
17·0—17·9	1	1	12	19	12	13	9	1	68	11
18·0—18·9	—	1	4	26	17	22	4	1	75	11
19·0—19·9	—	—	—	—	1	8	1	—	10	2
20·0—20·9	—	—	—	—	—	4	4	2	10	1
Totals	1	13	48	97	90	85	27	7	368	60

TABLE VII.

Grandparents and Grandchildren.

Grandchildren: Frontal Breadth. Unit = 1 division = ·0221 mm.

Grandparents: Frontal Breadth. Unit = 1 division of Ocular Micrometer = ·0221 mm.	*13·0–13·9*	*14·0–14·9*	*15·0–15·9*	*16·0–16·9*	*17·0–17·9*	Weighted Grandparents	Actual Number of Grandparents
14·0—14·9	3	4	1	—	—	8	1
15·0—15·9	—	12	5	2	—	19	4
16·0—16·9	3	46	50	30	2	131	14
17·0—17·9	2	23	44	18	—	87	7
18·0—18·9	—	8	8	12	3	31	3
19·0—19·9	—	—	8	5	2	15	1
Totals	8	93	116	67	7	291	30

TABLE VIII.

Parents and Offspring.

Offspring: Length of R. Antenna. Unit = 2 divisions of Ocular Micrometer = ·0442 mm.

Parents: Length of R. Antenna. Unit = 2 divisions of Ocular Micrometer = ·0442 mm.	24·0–25·9	26·0–27·9	28·0–29·9	30·0–31·9	32·0–33·9	34·0–35·9	36·0–37·9	38·0–39·9	40·0–41·9	42·0–43·9	44·0–45·9	46·0–47·9	48·0–49·9	50·0–51·9	Weighted Mothers	Number of Families
30·0—31·9	—	—	2	3	1	2	4	1	—	—	—	—	—	—	13	2
32·0—33·9	—	—	1	7	1	2	6	2	1	1	—	—	—	—	21	4
34·0—35·9	1	2	7	4	11	10	13	18	11	4	4	2	—	—	87	14
36·0—37·9	—	—	1	5	4	5	3	11	6	1	2	0	1	—	39	8
38·0—39·9	—	—	1	5	7	13	17	9	13	10	7	2	1	—	85	14
40·0—41·9	—	—	—	4	4	5	7	10	13	7	6	5	1	1	63	9
42·0—43·9	—	—	—	2	5	4	3	3	1	—	—	—	—	—	21	4
44·0—45·9	—	—	—	1	0	1	2	1	3	6	6	4	1	—	25	3
46·0—47·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0
48·0—49·9	—	—	—	—	—	—	—	1	1	1	1	—	—	—	4	1
50·0—51·9	—	—	—	—	—	—	—	—	—	2	5	2	1	—	10	1
Totals	1	2	12	31	33	42	55	56	51	33	31	15	5	1	368	60

TABLE IX.

Grandparents and Grandchildren.

Grandchildren: Length of R. Antenna. Unit = 2 divisions = ·0442 mm.

Grandparents: Length of R. Antenna. Unit = 2 divisions of Ocular Micrometer = ·0442 mm.	24·0–25·9	26·0–27·9	28·0–29·9	30·0–31·9	32·0–33·9	34·0–35·9	36·0–37·9	38·0–39·9	40·0–41·9	42·0–43·9	Weighted Grand-parents	Actual Number of Grand-parents
30·0—31·9	—	—	—	3	1	4	—	—	—	—	8	1
32·0—33·9	—	—	—	2	4	8	8	4	3	—	29	2
34·0—35·9	—	1	6	15	10	3	11	7	2	—	55	7
36·0—37·9	—	4	7	10	9	7	4	2	—	—	43	7
38·0—39·9	—	—	4	5	9	10	17	10	5	—	60	6
40·0—41·9	1	0	3	11	25	15	10	7	4	2	78	5
42·0—43·9	—	—	—	—	2	0	0	1	—	—	3	1
44·0—45·9	—	—	—	—	—	3	2	3	4	3	15	1
Totals	1	5	20	46	60	50	52	34	18	5	291	30

TABLE X.

Parents and Offspring.

Offspring: Ratio. Unit = 10 thousandths of Frontal Breadth.

Parents: Ratio $\left(\frac{CD}{AB}\right)$. Unit = 1 hundredth of Frontal Breadth.

	18·0–18·9	19·0–19·9	20·0–20·9	21·0–21·9	22·0–22·9	23·0–23·9	24·0–24·9	25·0–25·9	26·0–26·9	27·0–27·9	28·0–28·9	Weighted Mothers	Number of Families
18·0—18·9	—	—	—	—	1	6	—	—	—	—	—	7	1
19·0—19·9	—	—	—	—	—	—	—	—	—	—	—	0	0
20·0—20·9	1	2	6	15	8	4	2	—	—	—	—	38	7
21·0—21·9	—	4	7	9	13	18	13	4	—	—	—	68	10
22·0—22·9	—	4	2	13	17	26	21	14	7	—	—	104	18
23·0—23·9	—	1	3	7	16	8	18	17	5	1	—	76	14
24·0—24·9	—	—	—	6	7	7	9	14	4	1	—	48	7
25·0—25·9	—	—	—	—	1	1	12	4	4	1	4	27	3
Totals	1	11	18	50	63	70	75	53	20	3	4	368	60

TABLE XI.

Grandparents and Grandchildren.

Grandchildren: Ratio. Unit = 10 thousandths of Frontal Breadth.

Grandparents: Ratio. Unit = 1 hundredth of Frontal Breadth.

	18·0–18·9	19·0–19·9	20·0–20·9	21·0–21·9	22·0–22·9	23·0–23·9	24·0–24·9	25·0–25·9	26·0–26·9	Weighted Grand-parents	Actual Number of Grand-parents
20·0—20·9	—	5	11	11	9	13	5	1	—	55	4
21·0—21·9	—	1	7	9	10	15	3	1	—	46	4
22·0—22·9	—	4	8	13	8	13	6	1	—	53	10
23·0—23·9	2	2	11	20	29	16	21	10	3	114	10
24·0—24·9	—	—	—	4	3	6	5	5	—	23	2
Totals	2	12	37	57	59	63	40	18	3	291	30

TABLE XVI.

Pairs of Brethren.

Frontal Breadth, 2nd generation (60 families, 368 individuals).

The 2nd Individual taken.

The 1st Individual taken. Unit = 1 division of Ocular Micrometer = ·0221 mm.

	12·0–12·9	*13·0–13·9*	*14·0–14·9*	*15·0–15·9*	*16·0–16·9*	*17·0–17·9*	*18·0–18·9*	*19·0–19·9*	Number of individuals involved
12·0—12·9	—	—	4	1	—	—	—	—	5
13·0—13·9	—	26	21	8	1	1	—	—	57
14·0—14·9	4	21	114	101	20	10	—	—	270
15·0—15·9	1	8	101	210	149	56	6	1	532
16·0—16·9	—	1	20	149	160	146	31	8	515
17·0—17·9	—	1	10	56	146	266	98	20	597
18·0—18·9	—	—	—	6	31	98	50	17	202
19·0—19·9	—	—	—	1	8	20	17	4	50
Totals	5	57	270	532	515	597	202	50	2228

TABLE XVII.

Pairs of Grandchildren.

Frontal Breadth, 3rd generation (30 sets of families, 291 individuals).

The 2nd Individual taken.

The 1st Individual taken. Unit = 1 division of Ocular Micrometer = ·0221 mm.

	13·0–13·9	*14·0–14·9*	*15·0–15·9*	*16·0–16·9*	*17·0–17·9*	Number of individuals involved
13·0—13·9	8	39	23	9	—	79
14·0—14·9	39	440	387	112	9	987
15·0—15·9	23	387	612	417	39	1478
16·0—16·9	9	112	417	518	66	1122
17·0—17·9	—	9	39	66	8	122
Totals	79	987	1478	1122	122	3788

TABLE XVIII.

Pairs of Brethren.

Length of R. Antenna, 2nd generation (60 families, 368 individuals).

The 2nd Individual taken.

The 1st Individual taken. Unit = 2 divisions of Ocular Micrometer = ·0442 mm.

	24·0–25·9	26·0–27·9	28·0–29·9	30·0–31·9	32·0–33·9	34·0–35·9	36·0–37·9	38·0–39·9	40·0–41·9	42·0–43·9	44·0–45·9	46·0–47·9	48·0–49·9	50·0–51·9	Number of individuals involved
24·0—25·9	—	—	1	1	1	—	—	—	—	—	—	—	—	—	3
26·0—27·9	—	2	6	4	2	—	—	—	—	—	—	—	—	—	14
28·0—29·9	1	6	8	23	14	4	5	5	2	1	1	1	—	—	71
30·0—31·9	1	4	23	50	35	13	13	4	3	1	1	—	—	—	148
32·0—33·9	1	2	14	35	38	36	22	17	12	3	1	1	—	—	182
34·0—35·9	—	—	4	13	36	56	54	31	16	3	1	—	—	—	214
36·0—37·9	—	—	5	13	22	54	66	59	44	24	15	10	2	1	315
38·0—39·9	—	—	5	4	17	31	59	64	76	19	20	14	3	2	314
40·0—41·9	—	—	2	3	12	16	44	76	78	43	30	14	5	1	324
42·0—43·9	—	—	1	1	3	3	24	19	43	46	56	25	9	1	231
44·0—45·9	—	—	1	1	1	1	15	20	30	56	48	35	15	2	225
46·0—47·9	—	—	1	0	1	0	10	14	14	25	35	20	8	4	132
48·0—49·9	—	—	—	—	—	—	2	3	5	9	15	8	0	1	43
50·0—51·9	—	—	—	—	—	—	1	2	1	1	2	4	1	—	12
Totals	3	14	71	148	182	214	315	314	324	231	225	132	43	12	2228

TABLE XIX.

Pairs of Grandchildren.

Length of R. Antenna, 3rd generation (30 sets of families, 291 individuals).

The 2nd Individual taken.

The 1st Individual taken. Unit = 2 divisions of Ocular Micrometer = ·0442 mm.

	24·0–25·9	26·0–27·9	28·0–29·9	30·0–31·9	32·0–33·9	34·0–35·9	36·0–37·9	38·0–39·9	40·0–41·9	42·0–43·9	Number of individuals involved
24·0—25·9	—	—	—	3	3	3	—	—	—	—	9
26·0—27·9	—	6	5	9	4	6	3	—	—	—	33
28·0—29·9	—	5	22	75	33	21	13	6	5	—	180
30·0—31·9	3	9	75	194	110	74	33	23	11	4	536
32·0—33·9	3	4	33	110	178	134	118	70	36	10	696
34·0—35·9	3	6	21	74	134	128	156	90	62	17	691
36·0—37·9	—	3	13	33	118	156	214	152	85	12	786
38·0—39·9	—	—	6	23	70	90	152	74	53	17	485
40·0—41·9	—	—	5	11	36	62	85	53	24	14	290
42·0—43·9	—	—	—	4	10	17	12	17	14	8	82
Totals	9	33	180	536	696	691	786	485	290	82	3788

TABLE XX.

Pairs of Brethren.

Ratio, 3rd generation (60 families, 368 individuals).

The 2nd Individual taken.

The 1st Individual taken. Unit = 1 hundredth of Frontal Breadth.

	18·0–18·9	*19·0–19·9*	*20·0–20·9*	*21·0–21·9*	*22·0–22·9*	*23·0–23·9*	*24·0–24·9*	*25·0–25·9*	*26·0–26·9*	*27·0–27·9*	*28·0–28·9*	Number of individuals involved
18·0—18·9	—	—	2	1	—	—	—	—	—	—	—	3
19·0—19·9	—	4	19	18	7	6	5	1	2	—	—	62
20·0—20·9	2	19	30	32	10	1	5	—	—	—	—	99
21·0—21·9	1	18	32	62	71	30	21	5	3	—	—	243
22·0—22·9	—	7	10	71	122	83	41	14	16	0	4	368
23·0—23·9	—	6	1	30	83	136	91	57	27	1	—	432
24·0—24·9	—	5	5	21	41	91	130	94	39	10	12	448
25·0—25·9	—	1	0	5	14	57	94	118	41	7	4	341
26·0—26·9	—	2	0	3	16	27	39	41	18	2	16	164
27·0—27·9	—	—	—	—	—	1	10	7	2	—	—	20
28·0—28·9	—	—	—	—	4	0	12	4	16	0	12	48
Totals	3	62	99	243	368	432	448	341	164	20	48	2228

TABLE XXI.

Pairs of Grandchildren.

Ratio, 3rd generation (30 sets of families, 291 individuals).

The 2nd Individual taken.

The 1st Individual taken. Unit = 1 hundredth of Frontal Breadth.

	18·0–18·9	*19·0–19·9*	*20·0–20·9*	*21·0–21·9*	*22·0–22·9*	*23·0–23·9*	*24·0–24·9*	*25·0–25·9*	*26·0–26·9*	Number of individuals involved
18·0—18·9	—	1	0	3	6	0	0	1	—	11
19·0—19·9	1	26	37	42	14	11	1	—	—	132
20·0—20·9	—	37	88	94	89	59	37	6	2	412
21·0—21·9	3	42	94	148	155	134	65	34	2	677
22·0—22·9	6	14	89	155	202	202	129	45	7	849
23·0—23·9	—	11	59	134	202	266	154	53	4	883
24·0—24·9	—	1	37	65	129	154	100	56	12	554
25·0—25·9	1	0	6	34	45	53	56	34	7	236
26·0—26·9	—	—	2	2	7	4	12	7	—	34
Totals	11	132	412	677	849	883	554	236	34	3788

TABLES FOR TESTING THE GOODNESS OF FIT OF THEORY TO OBSERVATION.

By W. PALIN ELDERTON, *Actuary.*

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On the Test for Random Sampling.

Any theoretical description by means of curve or series is *ceteris paribus* admissible as a graduation of a given set of frequency observations, provided the observed values do not differ from the values provided by this theory by more than the reasonable deviations due to random sampling. There may be utilitarian reasons (e.g. relative fewness of descriptive constants, or their easy calculation) or philosophical reasons (e.g. general theories as to the nature and distribution of causes producing frequency phenomena) why we should adopt one theoretical description rather than another, but apart from such reasons that theoretical description is best, which describes the observed frequencies with the "greatest probability." By "describing the observed frequencies with the greatest probability" we understand a good although conventional test of fitness. Suppose the theoretical description of the frequencies to be the actual distribution of the whole population; we ask in how many cases per 100 in a series of random samplings should we differ from the theoretical distribution by as wide a system of deviations as that observed, or by a still wider system? In other words we want to find out the probability P that in random sampling deviation-systems as great as or greater than that actually observed will arise. This point has been dealt with in a paper by Professor K. Pearson published in the *Philosophical Magazine**, and it is there shown that if there be $n' = n + 1$ frequency groups in the series, and m_r and m_r' be the theoretical and observed frequencies in any group, it is necessary to find

$$\chi^2 = S\left\{\frac{(m_r - m_r')^2}{m_r}\right\} = \text{sum}\left(\frac{\left\{\begin{array}{c}\text{squares of differences of theoretical}\\ \text{and observed frequencies}\end{array}\right\}}{\text{theoretical frequency}}\right),$$

* On the Criterion that a given System of Deviations from the Probable in the case of a Correlated System of Variables is such that it can be reasonably supposed to have arisen from Random Sampling, Vol. L. pp. 157—175, July, 1900.

and that P will then be calculated from:

$$P=\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty}e^{-\frac{1}{2}x^2}\,dx+\sqrt{\frac{2}{\pi}}\,e^{-\frac{1}{2}\chi^2}\left(\frac{\chi}{1}+\frac{\chi^3}{1.3}+\frac{\chi^5}{1.3.5}+\ldots+\frac{\chi^{n'-3}}{1.3.5\ldots(n'-3)}\right)$$

if n' be even, and from:

$$P=e^{-\frac{1}{2}\chi^2}\left(1+\frac{\chi^2}{2}+\frac{\chi^4}{2.4}+\frac{\chi^6}{2.4.6}+\ldots+\frac{\chi^{n'-3}}{2.4.6\ldots(n'-3)}\right)$$

if n' be odd.

Now although χ^2 can be found quite easily without any special mathematical knowledge, the calculation of P from the above formulæ is very troublesome. But it is quite clear that some test of the above kind is absolutely needful in all biometric enquiries in which we wish to test theory against observation. In the paper referred to a small table for P in terms of n' and χ^2 was given, but this table beside being far from extensive enough for actual practice, was based in some entries on values of the probability integral which had not been calculated by the use of higher differences. The present Table I. is an attempt to provide a more extensive and accurate system of values for P. It gives the values of P for $n'=3$ to 30 and from $\chi^2=1$ to 30 by units and from $\chi^2=30$ to 70 by tens.

Method of Calculating Tables.

In order to simplify the work of calculating P for values lying outside the range of this table, or in cases where interpolation would not give sufficiently accurate results a series of additional tables are given which were used in the calculation of Table I. Thus Table II. gives the values of $\log\left(\chi\sqrt{\frac{2}{\pi}}\,e^{-\frac{1}{2}\chi^2}\right)$ and $\log\left(e^{-\frac{1}{2}\chi^2}\right)$ to eight figures. Table V. gives $\log e^{-\frac{1}{2}}$ and $\log\sqrt{\frac{2}{\pi}}$ to ten figures*. Table III. gives the cologarithms of $n(n-2)(n-4)\ldots1$ (or 2) needed for the coefficients of the powers of χ to eight figures. Table IV. gives the values of $\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty}e^{-\frac{1}{2}x^2}\,dx$ for $\chi^2=1$ to 30 to eight figures, i.e. as long as it is practically sensible. Further values of this integral may be deduced from the tables for $\frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}\,dt$, which are given for $t=0$ to 4·80 to eleven places of decimals for the higher values in Czuber's *Theorie der Beobachtungsfehler*, Leipzig, 1891.

In calculating the tables Erskine Scott's 10-Figure Logarithms and Filipowski's 7-Figure Antilogarithms were used. The method of calculation was, briefly, as follows. Tables were first made of $\log\left(\sqrt{\frac{2}{\pi}}\,e^{-\frac{1}{2}\chi^2}\right)$ and $\log e^{-\frac{1}{2}\chi^2}$ by continuous

* Thus incidentally the ordinates of the normal probability curve, $y=\frac{1}{2}\sqrt{\frac{2}{\pi}}\,e^{-\frac{1}{2}x^2}$, are given for the squares of the abscissæ.

addition and after adding $\log \chi$ to the former, the resulting figures were reduced from ten to eight places of decimals so as to avoid the error that would arise from the accumulation of small differences in the eleventh place in the value of $\log e^{-\frac{1}{2}}$. These tables were carefully checked by addition and by examining every tenth value in the continuous work. The table of $\operatorname{colog}[n]\left(=\log\frac{1}{n(n-2)(n-4)\ldots}\right)$ was originally calculated to ten places. The only other auxiliary table required was for $\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi$, and these values were calculated to seven places of decimals by second differences from a table of values of $\frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}\,dt$*. It was quite safe to omit third differences. The values of P were then calculated from formulæ (i) and (ii) given above. In making the table, to find χ^{2s}, a column of $s\log\chi^2$ was first set up, and then by means of four moveable slips of paper (two for n' even and two for n' odd) a second column calculated giving the sum of $s\log\chi^2$, $\operatorname{colog}(2s+1)$ and $\log\left(\sqrt{\frac{2}{\pi}}\chi e^{-\frac{1}{2}\chi^2}\right)$. These figures were checked by addition. The use of slips with $\operatorname{colog}(n)$ written on them saved a very large amount of copying. The antilogarithms of the items of the second column were then put in a third column and the values of $\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\,d\chi$ having been written at the top of it, the figures given in Table I. were found by continuous addition. The values for n' even were calculated in like manner. The numbers obtained were tested when possible against those originally published in the *Phil. Mag.* and against a few additional values calculated by Miss M. A. Lewenz. The work was of course checked at every stage, but when the table was completed the second differences in each column were examined and found to run smoothly. The like method of differences was appealed to in the case of discrepancies between the short table and the present table, which were not due to the approximate value taken for the probability integral. It is hoped that the table as it now stands is substantially free from error.

In using the present method of testing goodness of fit it is essential to bear in mind a warning given in the paper in the *Phil. Mag.* referred to above (footnote, p. 164): "A theoretical probability curve without limited range will never at the extreme tails exactly fit observation. The difficulty is obvious where the observations go by units and the theory by fractions. We ought to take our final theoretical groups to cover as much of the tail area as amounts to at least a unit of frequency in such cases."

Further we ought to be careful to read the corresponding *areas* of the frequency curve and not merely the mid-ordinates, when we have not a great number of groups, or when, although the groups are numerous, the frequency is very skew.

* The Table in Galloway's *Treatise on Probabilities* was the one actually used.

Illustration of use of Tables.

In the table below we have the distribution of the cephalic index in 900 skulls of modern Bavarian peasants. The frequency is given in the second column. In the third column we have the distribution as indicated by the normal curve of errors. Is this a reasonable description of the series of measurements? In the fourth column are given the values of $m_r - m_r'$ and in the fifth those of $(m_r - m_r')^2/m_r$. The resulting value of χ^2 is 18·36 and $n' = 24$. Table I. gives us: $n' = 24$, $\chi^2 = 18$, $P = \cdot 757489$, and $\chi^2 = 19$, $P = \cdot 701224$. Hence the required probability is nearly ·737, or roughly in every three cases out of four a random sampling would lead to a system of deviations diverging more widely from theory. Thus the fit may be considered a good one.

Cephalic Index of Bavarian Skulls.

Index	Observed	Calculated*	$m_r - m_r'$	$\frac{(m_r - m_r')^2}{m_r}$†
Under 71·5	2	1	− 1	1
72	0	1	+ 1	1
73	2·5	1·5	− 1	·67
74	1·5	3·5	+ 2	1·14
75	3·5	7·5	+ 4	2·13
76	12·5	13·5	+ 1	·07
77	17	23	+ 6	1·57
78	37	35·5	− 1·5	·06
79	55	52·5	− 2·5	·12
80	71·5	69·5	− 2	·06
81	82	86	+ 4	·19
82	116	98·5	−17·5	3·11
83	98	103	+ 5	·24
84	107	99·5	− 7·5	·57
85	82	88·5	+ 6·5	·48
86	74	72	− 2	·06
87	58	54	− 4	·30
88	34·5	37·5	+ 3	·24
89	19	23·5	+ 4·5	·86
90	10	14	+ 4	1·14
91	8	7·5	− 0·5	·03
92	3	3·5	+ 0·5	·07
93	1·5	2	+ 0·5	·125
Over 93·5	4·5	2	− 2·5	3·125
Totals	900	900	0	$\chi^2 = 18{\cdot}36$

* The calculated values are given to the nearest half skull because the observed values only run to this unit.

† The numbers in the fifth column were obtained from the squares of those in the fourth by dividing them by the corresponding numbers in the third. The squaring is at once done from *Barlow's Tables* and the division to the accuracy required by *Crelle's Rechentafeln*. Both these books are indispensable to biometricians.

TABLE I.

χ^2	$n'=3$	$n'=4$	$n'=5$	$n'=6$	$n'=7$	$n'=8$	$n'=9$	$n'=10$	$n'=11$
1	·606531	·801253	·909796	·962566	·985612	·994829	·998249	·999438	·999828
2	·367879	·572407	·735759	·849146	·919699	·959840	·981012	·991468	·996340
3	·223130	·391625	·557825	·699986	·808847	·885002	·934357	·964295	·981424
4	·135335	·261464	·406006	·549416	·676676	·779778	·857123	·911413	·947347
5	·082085	·171797	·287298	·415880	·543813	·659963	·757576	·834308	·891178
6	·049787	·111610	·199148	·306219	·423190	·539750	·647232	·739919	·815263
7	·030197	·071897	·135888	·220640	·320847	·428880	·536632	·637119	·725444
8	·018316	·046012	·091578	·156236	·238103	·332594	·433470	·534146	·628837
9	·011109	·029291	·061099	·109064	·173578	·252656	·342296	·437274	·532104
10	·006738	·018566	·040428	·075235	·124652	·188573	·265026	·350485	·440493
11	·004087	·011726	·026564	·051380	·088376	·138619	·201699	·275709	·357518
12	·002479	·007383	·017351	·034787	·061969	·100558	·151204	·213308	·285057
13	·001503	·004637	·011276	·023379	·043036	·072109	·111850	·162607	·223672
14	·000912	·002905	·007295	·015609	·029636	·051181	·081765	·122325	·172992
15	·000553	·001817	·004701	·010363	·020256	·036000	·059145	·090937	·132061
16	·000335	·001134	·003019	·006844	·013754	·025116	·042380	·066881	·099632
17	·000203	·000707	·001933	·004500	·009283	·017396	·030109	·048716	·074364
18	·000123	·000440	·001234	·002947	·006232	·011970	·021226	·035174	·054964
19	·000075	·000273	·000786	·001922	·004164	·008187	·014860	·025193	·040263
20	·000045	·000170	·000499	·001250	·002769	·005570	·010336	·017913	·029253
21	·000028	·000105	·000317	·000810	·001835	·003770	·007147	·012650	·021093
22	·000017	·000065	·000200	·000524	·001211	·002541	·004916	·008880	·015105
23	·000010	·000040	·000127	·000338	·000796	·001705	·003364	·006197	·010747
24	·000006	·000025	·000080	·000217	·000522	·001139	·002292	·004301	·007600
25	·000004	·000016	·000050	·000139	·000341	·000759	·001554	·002971	·005345
26	·000002	·000010	·000032	·000090	·000223	·000504	·001050	·002043	·003740
27	·000001	·000006	·000020	·000057	·000145	·000333	·000707	·001399	·002604
28	·000001	·000004	·000012	·000037	·000094	·000220	·000474	·000954	·001805
29	·000001	·000002	·000008	·000023	·000061	·000145	·000317	·000648	·001246
30	·000000	·000001	·000005	·000015	·000039	·000095	·000211	·000439	·000857
40	·000000	·000000	·000000	·000000	·000001	·000001	·000003	·000008	·000017
50	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000
60	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000
70	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000

TABLE I.—*continued.*

χ^2	$n'=12$	$n'=13$	$n'=14$	$n'=15$	$n'=16$	$n'=17$	$n'=18$	$n'=19$	$n'=20$
1	·999950	·999986	·999997	·999999	1·	1·	1·	1·	1·
2	·998496	·999406	·999774	·999917	·999970	·999990	·999997	·999999	1·
3	·990726	·995544	·997934	·999074	·999598	·999830	·999931	·999972	·999989
4	·969917	·983436	·991191	·995466	·997737	·998903	·999483	·999763	·999894
5	·931167	·957979	·975193	·985813	·992127	·995754	·997771	·998860	·999431
6	·873365	·916082	·946153	·966491	·979749	·988095	·993187	·996197	·997929
7	·799073	·857613	·902151	·934711	·957650	·973260	·983549	·990125	·994213
8	·713304	·785131	·843601	·889327	·923783	·948867	·966547	·978637	·986671
9	·621892	·702931	·772943	·831051	·877517	·913414	·940261	·959743	·973479
10	·530387	·615960	·693934	·762183	·819739	·866628	·903610	·931906	·952946
11	·443263	·528919	·610817	·686036	·752594	·809485	·856564	·894357	·923839
12	·362642	·445680	·527643	·606303	·679028	·743980	·800136	·847237	·885624
13	·293326	·369041	·447812	·526524	·602298	·672758	·736186	·791573	·838571
14	·232993	·300708	·373844	·449711	·525529	·598714	·667102	·729091	·783691
15	·182498	·241436	·307354	·378154	·451418	·524638	·595482	·661967	·722598
16	·141130	·191236	·249129	·313374	·382051	·452961	·523834	·592547	·657277
17	·107876	·149597	·199304	·256178	·318864	·385597	·454366	·523105	·589868
18	·081581	·115691	·157520	·206781	·262666	·323897	·388841	·455653	·522438
19	·061094	·088529	·123104	·164949	·213734	·268663	·328532	·391823	·456836
20	·045341	·067086	·095210	·130141	·171932	·220220	·274229	·332819	·394578
21	·033371	·050380	·072929	·101632	·136830	·178510	·226291	·279413	·336801
22	·024374	·037520	·055362	·078614	·107804	·143191	·184719	·231985	·284256
23	·017676	·027726	·041677	·060270	·084140	·113735	·149251	·190590	·237342
24	·012733	·020341	·031130	·045822	·065093	·089504	·119435	·155028	·196152
25	·009117	·014822	·023084	·034566	·049943	·069824	·094710	·124915	·160542
26	·006490	·010734	·017001	·025887	·038023	·054028	·074461	·099758	·130189
27	·004595	·007727	·012441	·019254	·028736	·041483	·058068	·078995	·104653
28	·003238	·005532	·009050	·014228	·021569	·031620	·044938	·062055	·083428
29	·002270	·003940	·006546	·010450	·016085	·023936	·034526	·048379	·065985
30	·001585	·002792	·004710	·007632	·011921	·018002	·026345	·037446	·051798
40	·000036	·000072	·000138	·000255	·000453	·000778	·001294	·002087	·003272
50	·000001	·000001	·000003	·000006	·000012	·000023	·000042	·000075	·000131
60	·000000	·000000	·000000	·000000	·000000	·000001	·000001	·000002	·000004
70	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000	·000000

TABLE I.—*continued.*

χ^2	$n'=21$	$n'=22$	$n'=23$	$n'=24$	$n'=25$	$n'=26$	$n'=27$	$n'=28$	$n'=29$	$n'=30$
1	1·	1·	1·	1·	1·	1·	1·	1·	1·	1·
2	1·	1·	1·	1·	1·	1·	1·	1·	1·	1·
3	·999996	·999998	·999999	1·	1·	1·	1·	1·	1·	1·
4	·999954	·999980	·999992	·999997	·999999	1·	1·	1·	1·	1·
5	·999722	·999868	·999939	·999972	·999987	·999994	·999998	·999999	1·	1·
6	·998898	·999427	·999708	·999855	·999929	·999966	·999984	·999993	·999997	·999999
7	·996685	·998142	·998980	·999452	·999711	·999851	·999924	·999962	·999981	·999991
8	·991868	·995143	·997160	·998371	·999085	·999494	·999726	·999853	·999924	·999960
9	·982907	·989214	·993331	·995957	·997595	·998596	·999194	·999546	·999748	·999863
10	·968171	·978912	·986304	·991277	·994547	·996653	·997981	·998803	·999302	·999599
11	·946223	·962787	·974749	·983189	·989012	·992946	·995549	·997239	·998315	·998988
12	·916076	·939617	·957379	·970470	·979908	·986567	·991173	·994294	·996372	·997728
13	·877384	·908624	·933161	·951990	·966121	·976501	·983974	·989247	·992900	·995384
14	·830496	·869599	·901479	·926871	·946650	·961732	·973000	·981254	·987189	·991377
15	·776408	·822952	·862238	·894634	·920759	·941383	·957334	·969432	·978436	·985015
16	·716624	·769650	·815886	·855268	·888076	·914828	·936203	·952947	·965819	·975536
17	·652974	·711106	·763362	·809251	·848662	·881793	·909083	·931122	·948589	·962181
18	·587408	·649004	·705988	·757489	·803008	·842390	·875773	·903519	·926149	·944272
19	·521826	·585140	·645328	·701224	·751990	·797120	·836430	·870001	·898136	·921288
20	·457930	·521261	·583040	·641912	·696776	·746825	·791556	·830756	·864464	·892927
21	·397132	·458944	·520738	·581087	·638725	·692609	·741964	·786288	·825349	·859149
22	·340511	·399510	·459889	·520252	·579267	·635744	·688697	·737377	·781291	·820189
23	·288795	·343979	·401730	·460771	·519798	·577564	·632947	·685013	·733041	·776543
24	·242392	·293058	·347229	·403808	·461597	·519373	·575965	·630316	·681535	·728932
25	·201431	·247164	·297075	·350285	·405760	·462373	·518975	·574462	·627835	·678248
26	·165812	·206449	·251682	·300866	·353165	·407598	·463105	·518600	·573045	·625491
27	·135264	·170853	·211226	·255967	·304453	·355884	·409333	·463794	·518247	·571705
28	·109399	·140151	·175681	·215781	·260040	·307853	·358458	·410973	·464447	·517913
29	·087759	·114002	·144861	·180310	·220131	·263916	·311082	·360899	·412528	·465066
30	·069854	·091988	·118464	·149402	·184752	·224289	·267611	·314154	·363218	·414004
40	·004995	·007437	·010812	·015369	·021387	·029164	·039012	·051237	·066128	·083937
50	·000221	·000365	·000586	·000921	·001416	·002131	·003144	·004551	·006467	·009032
60	·000007	·000013	·000022	·000038	·000064	·000104	·000168	·000264	·000407	·000618
70	·000000	·000000	·000001	·000001	·000002	·000004	·000007	·000011	·000019	·000030

TABLE II.

χ^2	$\log\left\{\chi\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\right\}$	$\log e^{-\frac{1}{2}\chi^2}$	χ^2	$\log\left\{\chi\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\chi^2}\right\}$	$\log e^{-\frac{1}{2}\chi^2}$
1	$\bar{1}\cdot68479282$	$\bar{1}\cdot78285276$	51	$\bar{11}\cdot68121586$	$\bar{12}\cdot92549071$
2	$\bar{1}\cdot61816058$	$\bar{1}\cdot56570552$	52	$\bar{11}\cdot46828520$	$\bar{12}\cdot70834347$
3	$\bar{1}\cdot48905897$	$\bar{1}\cdot34855828$	53	$\bar{11}\cdot25527422$	$\bar{12}\cdot49119623$
4	$\bar{1}\cdot33438109$	$\bar{1}\cdot13141104$	54	$\bar{11}\cdot04218593$	$\bar{12}\cdot27404899$
5	$\bar{1}\cdot16568886$	$\bar{2}\cdot91426380$	55	$\bar{12}\cdot82902315$	$\bar{12}\cdot05690175$
6	$\bar{2}\cdot98813224$	$\bar{2}\cdot69711655$	56	$\bar{12}\cdot61578858$	$\bar{13}\cdot83975450$
7	$\bar{2}\cdot80445839$	$\bar{2}\cdot47996931$	57	$\bar{12}\cdot40248475$	$\bar{13}\cdot62260726$
8	$\bar{2}\cdot61630713$	$\bar{2}\cdot26282207$	58	$\bar{12}\cdot18911408$	$\bar{13}\cdot40546002$
9	$\bar{2}\cdot42473615$	$\bar{2}\cdot04567483$	59	$\bar{13}\cdot97567885$	$\bar{13}\cdot18831278$
10	$\bar{2}\cdot23046765$	$\bar{3}\cdot82852759$	60	$\bar{13}\cdot76218123$	$\bar{14}\cdot97116554$
11	$\bar{2}\cdot03401675$	$\bar{3}\cdot61138035$	61	$\bar{13}\cdot54862328$	$\bar{14}\cdot75401830$
12	$\bar{3}\cdot83576379$	$\bar{3}\cdot39423311$	62	$\bar{13}\cdot33500696$	$\bar{14}\cdot53687106$
13	$\bar{3}\cdot63599760$	$\bar{3}\cdot17708587$	63	$\bar{13}\cdot12133415$	$\bar{14}\cdot31972382$
14	$\bar{3}\cdot43494271$	$\bar{4}\cdot95993863$	64	$\bar{14}\cdot90760662$	$\bar{14}\cdot10257658$
15	$\bar{3}\cdot23277708$	$\bar{4}\cdot74279139$	65	$\bar{14}\cdot69382607$	$\bar{15}\cdot88542934$
16	$\bar{3}\cdot02964420$	$\bar{4}\cdot52564414$	66	$\bar{14}\cdot48099412$	$\bar{15}\cdot66828209$
17	$\bar{4}\cdot82566143$	$\bar{4}\cdot30849690$	67	$\bar{14}\cdot26611232$	$\bar{15}\cdot45113485$
18	$\bar{4}\cdot62092598$	$\bar{4}\cdot09134966$	68	$\bar{14}\cdot05218213$	$\bar{15}\cdot23398761$
19	$\bar{4}\cdot41551928$	$\bar{5}\cdot87420242$	69	$\bar{15}\cdot83820498$	$\bar{15}\cdot01684037$
20	$\bar{4}\cdot20951024$	$\bar{5}\cdot65705518$	70	$\bar{15}\cdot62418221$	$\bar{16}\cdot79969313$
21	$\bar{4}\cdot00295765$	$\bar{5}\cdot43990794$	71	$\bar{15}\cdot41011512$	$\bar{16}\cdot58254589$
22	$\bar{5}\cdot79591210$	$\bar{5}\cdot22276070$	72	$\bar{15}\cdot19600496$	$\bar{16}\cdot36539865$
23	$\bar{5}\cdot58841744$	$\bar{5}\cdot00561346$	73	$\bar{16}\cdot98185290$	$\bar{16}\cdot14825141$
24	$\bar{5}\cdot38051190$	$\bar{6}\ 78846622$	74	$\bar{16}\cdot76766009$	$\bar{17}\cdot93110417$
25	$\bar{5}\cdot17222904$	$\bar{6}\cdot57131898$	75	$\bar{16}\cdot55342762$	$\bar{17}\cdot71395693$
26	$\bar{6}\cdot96359847$	$\bar{6}\cdot35417173$	76	$\bar{16}\cdot33915654$	$\bar{17}\cdot49680968$
27	$\bar{6}\cdot75464644$	$\bar{6}\cdot13702449$	77	$\bar{16}\cdot12484787$	$\bar{17}\cdot27966244$
28	$\bar{6}\cdot54539633$	$\bar{7}\cdot91987725$	78	$\bar{17}\cdot91050256$	$\bar{17}\cdot06251520$
29	$\bar{6}\cdot33586907$	$\bar{7}\cdot70273001$	79	$\bar{17}\cdot69612157$	$\bar{18}\cdot84536796$
30	$\bar{6}\cdot12608346$	$\bar{7}\cdot48558277$	80	$\bar{17}\cdot48170578$	$\bar{18}\cdot62822072$
31	$\bar{7}\cdot91605644$	$\bar{7}\cdot26843553$	81	$\bar{17}\cdot26725605$	$\bar{18}\cdot41107348$
32	$\bar{7}\cdot70580334$	$\bar{7}\cdot05128829$	82	$\bar{17}\cdot05277323$	$\bar{18}\cdot19392624$
33	$\bar{7}\cdot49533808$	$\bar{8}\cdot83414105$	83	$\bar{18}\cdot83825810$	$\bar{19}\cdot97677900$
34	$\bar{7}\cdot28467333$	$\bar{8}\cdot61699381$	84	$\bar{18}\cdot62371146$	$\bar{19}\cdot75963176$
35	$\bar{7}\cdot07382065$	$\bar{8}\cdot39984657$	85	$\bar{18}\cdot40913404$	$\bar{19}\cdot54248452$
36	$\bar{8}\cdot86279064$	$\bar{8}\cdot18269932$	86	$\bar{18}\cdot19452656$	$\bar{19}\cdot32533727$
37	$\bar{8}\cdot65159301$	$\bar{9}\cdot96555208$	87	$\bar{19}\cdot97988972$	$\bar{19}\cdot10819003$
38	$\bar{8}\cdot44023670$	$\bar{9}\cdot74840484$	88	$\bar{19}\cdot76522419$	$\bar{20}\cdot89104279$
39	$\bar{8}\cdot22872997$	$\bar{9}\cdot53125760$	89	$\bar{19}\cdot55053062$	$\bar{20}\cdot67389555$
40	$\bar{8}\cdot01708042$	$\bar{9}\cdot31411036$	90	$\bar{19}\cdot33580963$	$\bar{20}\cdot45674831$
41	$\bar{9}\cdot80529511$	$\bar{9}\cdot09696312$	91	$\bar{19}\cdot12106183$	$\bar{20}\cdot23960107$
42	$\bar{9}\cdot59338058$	$\bar{10}\cdot87981588$	92	$\bar{20}\cdot90628780$	$\bar{20}\cdot02245383$
43	$\bar{9}\cdot38134293$	$\bar{10}\cdot66266864$	93	$\bar{20}\cdot69148812$	$\bar{21}\cdot80530659$
44	$\bar{9}\cdot16918780$	$\bar{10}\cdot44552140$	94	$\bar{20}\cdot47666333$	$\bar{21}\cdot58815935$
45	$\bar{10}\cdot95692047$	$\bar{10}\cdot22837416$	95	$\bar{20}\cdot26181397$	$\bar{21}\cdot37101211$
46	$\bar{10}\cdot74454589$	$\bar{10}\cdot01122691$	96	$\bar{20}\cdot04694054$	$\bar{21}\cdot15386486$
47	$\bar{10}\cdot53206866$	$\bar{11}\cdot79407967$	97	$\bar{21}\cdot83204355$	$\bar{22}\cdot93671762$
48	$\bar{10}\cdot31949311$	$\bar{11}\cdot57693243$	98	$\bar{21}\cdot61712348$	$\bar{22}\cdot71957038$
49	$\bar{10}\cdot10682329$	$\bar{11}\cdot35978519$	99	$\bar{21}\cdot40218080$	$\bar{22}\cdot50242314$
50	$\bar{11}\cdot89406301$	$\bar{11}\cdot14263795$	100	$\bar{21}\cdot18721596$	$\bar{22}\cdot28527590$

TABLE III.

Table of colog $[n]$:—$[n] = n(n-2)(n-4)\ldots\ldots$

n odd nos.	colog $[n]$	n even nos.	colog $[n]$
1	·00000000	2	$\bar{1}$·69897000
3	$\bar{1}$·52287875	4	$\bar{1}$·09691001
5	$\bar{2}$·82390874	6	$\bar{2}$·31875876
7	$\bar{3}$·97881070	8	$\bar{3}$·41566878
9	$\bar{3}$·02456819	10	$\bar{4}$·41566878
11	$\bar{5}$·98317551	12	$\bar{5}$·33648753
13	$\bar{6}$·86923215	14	$\bar{6}$·19035949
15	$\bar{7}$·69314089	16	$\bar{8}$·98623951
17	$\bar{8}$·46269197	18	$\bar{9}$·73096701
19	$\bar{9}$·18393837	20	$\bar{10}$·42993701
21	$\bar{11}$·86171908	22	$\bar{11}$·08751433
23	$\bar{12}$·49999124	24	$\bar{13}$·70730309
25	$\bar{13}$·10205123	26	$\bar{14}$·29232974
27	$\bar{15}$·67068747	28	$\bar{16}$·84517171
29	$\bar{16}$·20828947	30	$\bar{17}$·36805045
31	$\bar{18}$·71692778	32	$\bar{19}$·86290048
33	$\bar{19}$·19841384	34	$\bar{20}$·33142156
35	$\bar{21}$·65434579	36	$\bar{22}$·77511906
37	$\bar{22}$·08614407	38	$\bar{23}$·19533546
39	$\bar{24}$·49507946	40	$\bar{25}$·59327547
41	$\bar{26}$·88229561	42	$\bar{27}$·97002618
43	$\bar{27}$·24882715	44	$\bar{28}$·32657350
45	$\bar{29}$·59561464	46	$\bar{30}$·66381567
47	$\bar{31}$·92351678	48	$\bar{32}$·98257443
49	$\bar{32}$·23332070	50	$\bar{33}$·28360443
51	$\bar{34}$·52575052	52	$\bar{35}$·56760109
53	$\bar{36}$ 80147465	54	$\bar{37}$·83520733
55	$\bar{37}$·06111196	56	$\bar{38}$·08701930
57	$\bar{39}$·30523711	58	$\bar{40}$·32359131
59	$\bar{41}$·53438510	60	$\bar{42}$·54544006
61	$\bar{43}$·74905526	62	$\bar{44}$·75304837
63	$\bar{45}$·94971471	64	$\bar{46}$·94686839
65	$\bar{46}$·13680135	66	$\bar{47}$·12732446
67	$\bar{48}$·31072655	68	$\bar{49}$·29481554
69	$\bar{50}$·47187746	70	$\bar{51}$·44971750
71	$\bar{52}$·62061911	72	$\bar{53}$·59238501
73	$\bar{54}$·75729625	74	$\bar{55}$·72315329
75	$\bar{56}$·88223499	76	$\bar{57}$·84233970
77	$\bar{58}$·99574426	78	$\bar{59}$·95024509
79	$\bar{59}$·09811717	80	$\bar{60}$·04715511
81	$\bar{61}$·18963215	82	$\bar{62}$·13334125
83	$\bar{63}$·27055406	84	$\bar{64}$·20906197
85	$\bar{65}$·34113514	86	$\bar{66}$·27456352
87	$\bar{67}$·40161588	88	$\bar{68}$·33008084
89	$\bar{69}$·45222588	90	$\bar{70}$·37583833
91	$\bar{71}$·49318448	92	$\bar{72}$·41205051
93	$\bar{73}$·52470154	94	$\bar{74}$·43892265
95	$\bar{75}$·54697793	96	$\bar{76}$·45665142
97	$\bar{77}$·56020620	98	$\bar{78}$·46542534
99	$\bar{79}$·56457100	100	$\bar{80}$·46542534

TABLE IV.

χ^2	$\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty} e^{-\frac{1}{2}x^2}\,dx$
1	·3173106
2	·1572992
3	·0832646
4	·0455003
5	·0253474
6	·0143060
7	·0081506
8	·0046776
9	·0026998
10	·0015654
11	·0009112
12	·0005321
13	·0003115
14	·0001828
15	·0001076
16	·0000634
17	·0000374
18	·0000221
19	·0000132
20	·0000078
21	·0000046
22	·0000027
23	·0000016
24	·0000011
25	·0000007
26	·0000004
27	·0000003
28	·0000002
29	·0000001
30	·0000000

TABLE V.

Function	Log. Function
$e^{-\frac{1}{2}}$	$\bar{1}$·7828527590
$\sqrt{\frac{2}{\pi}}$	$\bar{1}$·9019400615

THE EGG OF CUCULUS CANORUS.

AN ENQUIRY INTO THE DIMENSIONS OF THE CUCKOO'S EGG AND THE RELATION OF THE VARIATIONS TO THE SIZE OF THE EGGS OF THE FOSTER-PARENT, WITH NOTES ON COLORATION, &c.

By OSWALD H. LATTER.

[Received October 2, 1901.]

Introductory.

THE present paper is the outcome of an examination of 44 Cuckoo's eggs in the collections at the Charterhouse Museum. The results of this preliminary investigation were communicated to the Congress of South Eastern Natural History Associations, held in the summer of 1901 at Haslemere, under the auspices of the Haslemere Microscope and Natural History Society, of which I have the honour to be a member. Finding that 44 was far too small a number of eggs for my purpose I extended the series of measurements by including a large number obtained at the British Museum of Natural History, S. Kensington, and I may here take the opportunity of expressing my thanks to the Director, Prof. Ray Lankester, and Mr Ogilvie Grant for granting me permission to examine the fine series under their care, and also to Mr Baldwin Young of Sheffield, who kindly supplied measurements of six Cuckoo's eggs in his possession. The total number of Cuckoo's eggs measured and included in this enquiry is 243, of which 223 were known to have been deposited in the nests of 42 different species of birds, while the foster-parents of the remaining 20 were not ascertainable; these 20 have not been excluded from the calculations, for their effect upon the value of mean length and breadth is practically negligible. In dealing with coloration, a further 45 which were not accessible for purposes of measurement have been included, bringing the total to 288. All measurements were taken with sliding

callipers reading to millimetres, the decimal parts of a millimetre being estimated by eye aided with a strong lens. The dimensions measured are greatest length and greatest breadth.

It has been established by many observers that the female Cuckoo lays her egg upon the ground and then taking it in her beak puts it into the nest of the foster-parents of her offspring. An explanation is needed of the success which attends this imposition. Are the foster-parents deceived either by similarity of colouring or of size into fancying the Cuckoo's egg to be one of their own? or are they indifferent to these qualities? or are some small birds more expert than others in detecting fraud?

The theory which finds more favour than others is that put forward by Prof. A. Newton (*Dictionary of Birds*, p. 123); who, after mentioning the history of speculation on the matter, writes as follows:—"Everyone who has sufficiently studied the habits of animals will admit the tendency of some of these habits to become hereditary. That there is a reasonable probability of each Cuckow most commonly putting her eggs in the nest of the same species of bird, and of this habit being transmitted to her posterity, does not seem to be a very violent supposition. Without attributing any wonderful sagacity to her, it does not seem unlikely that the Cuckow which had once successfully foisted her egg on a Reed-wren or a Titlark should again seek for another Reed-wren's or another Titlark's nest (as the case may be).....and that she should continue her practice from one season to another..... Such a habit could hardly fail to become hereditary, so that the daughter of a Cuckow which always put her egg into a Reed-wren's, Titlark's, or Wagtail's nest, would do as did her mother ..... and it can hardly be questioned that the eggs of the daughter would more or less resemble those of her mother. Hence the supposition may be fairly regarded that the habit of laying a particular style of egg is also likely to become hereditary ..... The particular '*gens*' of Cuckow which inherited and transmitted the habit of depositing in the nest of any particular species of bird eggs having more or less resemblance to the eggs of that species would prosper most in those members of the '*gens*' where the likeness was strongest, and the other members would (*caeteris paribus*) in time be eliminated..... The operation of this kind of natural selection would be most needed in those cases where the species are not easily duped—that is in those cases which occur the least frequently. Here it is we find it, for observation shows that eggs of the Cuckow deposited in the nests of the Red-backed Shrike, of the Bunting, of the Red Start and of the Icterine Warbler approximate in their colouring to eggs of these species—species in whose nests the Cuckow rarely (in comparison with others) deposits eggs."

I must confess that I approached this investigation with decided scepticism as to the validity of Prof. Newton's theory. It is very doubtful whether the Cuckoo is aware that she has "successfully foisted her egg on a Reed-wren" or on any other bird: so far as is known she takes no further interest in the egg—it may

escape detection, it may be ejected. Moreover, the theory seems to demand that male Cuckoos should mate with female Cuckoos reared by foster-parents of the same species, or else that the inherited habits and characters of every female Cuckoo should follow only the female line of descent. For suppose a Reed-wren-reared female Cuckoo to mate with a Robin-reared male Cuckoo, then their offspring might be reasonably expected to inherit some characters from each parent and to possess mixed tendencies, some urging them to lay in Robins' nests and others in those of Reed-wrens, and, unless inheritance run only in the female line (or mating taking place only between individuals of like foster-parentage), the tendencies would get further mingled in each succeeding generation. This criticism appears to me to apply with equal force to Reh's theory of the intuitive selection of the nests of the species by which the Cuckoos themselves were reared. Further, there is very little, if any, evidence in support of the operation of natural selection in eliminating eggs that do not match those of the clutch into which they have been introduced. I have not come across any record of such badly-matched Cuckoo's eggs having been found ejected from the nest. Lastly, it is very difficult to conceive how perfection of colour-matching can have arisen by natural selection with relatively few opportunities for the working of this force; and, if the case be as Prof. Newton states, I certainly should have expected to find a large number of Cuckoo's eggs in the nests of those birds whose eggs had at length been so admirably copied.

Size-Matching.

In spite of these criticisms I am compelled by the results of my investigations to admit that I now believe Professor Newton's theory to be in the main correct: I will return to this point later. It will be seen by the appended summary of results and tables of measurements that the mean length (22·40 mm.) and mean breadth (16·54 mm.) of 243 Cuckoo's eggs are respectively greater than the mean lengths and breadths of the eggs of any of the four species [viz. *Anthus pratensis*, the Meadow Pipit; *Anthus trivialis*, the Tree Pipit; *Accentor modularis*, the Hedge Sparrow, and *Erithacus rubecula*, the Robin], of whose eggs I was able to measure a reasonable number. The range of length extends from 19·1 mm. to 25·0 mm.; that of breadth from 14·0 mm. to 18·8 mm.: the standard deviation (σ) of length being 1·058, and the coefficient of variation (C. of V.) of length 4·72, those of breadth being respectively 0·6496 and 3·93. Of the four species which serve as foster-parents and are here dealt with, the Meadow Pipit alone exceeds the Cuckoo in degree of variation, but it must be remembered that the number of measured eggs of Robins and other species is very much less than in the case of the Cuckoo, so that it is quite possible that the Cuckoo does not differ greatly from other birds in this respect*. It is however interesting to note that in

* Further, the Cuckoo's eggs were probably laid by about 200 separate hens, while the 74 Meadow Pipit's eggs, for example, are formed by 20 clutches or due to 20 hens only.

each of the five species length of egg is a far more variable dimension than breadth: this is probably due to uniformity of diameter of the oviduct, and it may also be of importance to the comfort of the female bird during the period of incubation, for an egg projecting far above its fellows in consequence of greater breadth would probably inconvenience the sitter.

My enquiry has thus resolved itself chiefly into an attempt to ascertain (1) if the eggs of Cuckoos deposited in the nests of any one species stand out as a set apart from Cuckoo's eggs deposited elsewhere; (2) if the same eggs depart from the rest in such a direction as to approximate in size to the eggs of that particular species of foster-parent. The method employed is to find the mean (M) length or breadth, as the case may be, thence to compute the standard deviation (σ) by the formula $\sigma^2 = \frac{\text{sum}\,(M-x)^2}{n}$, where x = the measurement of any one egg and n = the number of eggs measured: and then to find $\frac{100\,\sigma}{M}$, the coefficient of variation. To test whether any deviation is significant, M_r is taken as the mean of the whole race of Cuckoos and M_s the mean of Cuckoo's eggs found in the nest of any one species of foster-parent: the standard deviation (σ_s) of such eggs is also ascertained. The value of $M_r - M_s$ is then compared with that of $0{\cdot}67449\sqrt{\frac{\sigma_r^2}{n_1} + \frac{\sigma_s^2}{n_2}}$, where n_1 = total number of Cuckoo's eggs and n_2 = the number of Cuckoo's eggs in the nests of the species in question, which is the probable error of $M_r - M_s$ due to random sampling. If the value of $M_r - M_s$ be not at least 1·5 to 3 times as great as the value of the other expression the difference of M_r and M_s is not definitely significant. Referring now to the tabulated summary below, it will be seen that in the matter of length, the eggs of Meadow-Pipit-Cuckoos (to coin a convenient phrase), of Wagtail-Cuckoos (this includes all species of Wagtails, for their eggs are very similar), and of Robin-Cuckoos, do not differ significantly from those of the whole race of Cuckoos, but those of Hedge-Sparrow-Cuckoos, Tree-Pipit-Cuckoos and of Wren-Cuckoos certainly do present differences marking them out as distinct sets. On the other hand in the matter of breadth the differences are significant in the cases of Meadow-Pipit-Cuckoos, Hedge-Sparrow-Cuckoos and Wren-Cuckoos. It therefore seems highly probable that there are certain "gentes" of Cuckoos whose members being closely related lay eggs of somewhat similar dimensions and in the main confine their attentions each to its own particular variety of foster-parent. Breadth, as I have already pointed out, seems more likely than length to be a disturbing factor in the nest of the foster-parent if it in any marked way depart from the normal, and it is very remarkable to note how very low are four of the values of σ_s in the breadth tables, viz. 2·28 (Meadow-Pipit-Cuckoo), 2·58 (Wagtail-Cuckoo), 2·65 (Tree-Pipit-Cuckoo), 1·92 (Wren-Cuckoo).

Next, as to whether these sets differ from the main body in the sense of the particular species of foster-parent. In the Wren-Cuckoos this is so beyond

Summary.

Species	Number of Eggs	Mean (M)	Standard Deviation (σ)	Coefficient of Variation	Significance Test (Ratio of difference to its probable error)
Length **in millimetres**					
Cuculus canorus	243	22·4	1·0585	4·72	
C. canorus-Anthus pratensis (Meadow-Pipit-Cuckoo) ...	45	22·3	0·8933	4·00	difference not significant (1·1)
Anthus pratensis	74	19·7	1·2504	6·37	
C. canorus-Accentor modularis (Hedge-Sparrow-Cuckoo) ...	14	23·1	1·0116	4·37	difference *significant* (3·71)
Accentor modularis	26	20·0	0·8096	4·02	
C. canorus-Erithacus rubecula (Robin-Cuckoo)	16	22·5	0·6628	2·50	difference not significant (1·25)
Erithacus rubecula	57	20·2	0·8565	4·24	
C. canorus-Motacilla, sp. 4 (Wagtail-Cuckoos) ...	26	22·6	0·8783	3·88	difference not significant (1·25)
Wagtails estimated	—	19·9			
C. canorus-Anthus trivialis (Tree-Pipit-Cuckoo) ...	15	23·1	0·8504	3·68	difference *significant* (4·5)
Anthus trivialis	27	20·0	0·6978	3·489	
C. canorus-Troglodytes parvulus	15	21·1	0·7558	3·58	difference *significant* (9·3)
Troglodytes parvulus ...	Estimated from W. J. Gordon	17·7			
Breadth **in millimetres**					
Cuculus canorus	243	16·5	0·6496	3·93	
C. canorus-Anthus pratensis (Meadow-Pipit-Cuckoo) ...	45	16·7	0·3815	2·28	difference *significant* (3·4)
Anthus pratensis	74	14·6	0·5611	3·84	
C. canorus-Accentor modularis (Hedge-Sparrow-Cuckoo) ...	14	16·8	0·5161	3·07	difference *significant* (2·4)
Accentor modularis	26	14·7	0·4146	2·81	
C. canorus-Erithacus rubecula (Robin-Cuckoo)	16	16·4	0·5326	3·24	difference not significant (·96)
Erithacus rubecula	57	15·4	0·4771	3·09	
C. canorus-Motacilla, sp. 4 (Wagtail-Cuckoos) ...	26	16·6	0·4389	2·58	difference not significant (·93)
Wagtails estimated	—	14·9			
C. canorus-Anthus trivialis (Tree-Pipit-Cuckoo) ...	15	16·6	0·4397	2·65	difference not significant (·75)
Anthus trivialis	27	15·1	0·4488	2·97	
C. canorus-Troglodytes parvulus	15	15·8	0·3042	1·92	difference *significant* (12·3)
Troglodytes parvulus ...	Estimated from W. J. Gordon's "Our Country's Birds"	12·7			

doubt. Unfortunately I had not material sufficient to determine trustworthy means of length and breadth of Wren's eggs, but I have estimated them from measurements given *in inches* by W. J. Gordon in *Our Country's Birds*, and feel confident that no error of any importance exists in his figures, for the measurements given by him of other species' eggs approximate very closely with the means obtained by myself in each case. It will be seen that the egg of the Wren is far smaller than that of any other species with which we are dealing, and that the lengths and breadths of Wren-Cuckoos' eggs are very much less than those of other Cuckoos. For the other species where the differences are significant this sense of the variation is not clear so far as it concerns breadth, though it appears to be so in the matter of length: the two subjoined tables give the comparison.

Mean Length			
Hedge-Sparrow-Cuckoo	23·1	Hedge-Sparrow ...	20·1
Tree-Pipit-Cuckoo ...	23·1	Tree-Pipit	20·0
Meadow-Pipit-Cuckoo	22·3	Meadow-Pipit... ...	19·7
Wren-Cuckoo	21·1	Wren	17·7

Mean Breadth			
Hedge-Sparrow-Cuckoo	16·8	Tree-Pipit	15·1
Meadow-Pipit-Cuckoo	16·7	Hedge-Sparrow ...	14·7
Tree-Pipit-Cuckoo ...	16·6	Meadow-Pipit... ...	14·6
Wren-Cuckoo	15·8	Wren	12·7

Colour-matching.

As already stated, 288 Cuckoo's eggs were examined in this connection and compared with the eggs among which they had been deposited. In 39 cases the matching was extremely close, and in a further 109 there was a fair approximation, rendering the Cuckoo's egg more or less similar in appearance to those of the foster-parent, the two categories giving a total of 148 eggs more or less closely matched to their several clutches, or at any rate within the limits of colour-variations exhibited by the species in question. Reh has observed this last same phenomenon in nests of the Red-backed Shrike (*Lanius collaris*), and eggs of this description possess an especial interest, for they seem to afford very strong support to Prof. Newton's theory. It is a fairly frequent occurrence to find in the nests of birds whose eggs exhibit considerable variation of colour, e.g. Meadow-Pipit, Tree-Pipit and Reed-Warbler, a Cuckoo's egg, not resembling the particular clutch in which it occurs but which would match eggs of another clutch of the same species. On the other hand, I found but three instances of Cuckoo's eggs with a close resemblance to eggs of any one species being deposited elsewhere than in the nests of that species, viz. egg No. 152 found in nest of Lesser Whitethroat is a good match to a Meadow-Pipit's eggs; No. 173 in nest

of Chaffinch matches Wagtail's eggs; No. 187 in Nightingale's nest matches House-Sparrow's (very variable) eggs.

Colour-matching.

Good		Not matched nor within limits of colour-variation of egg	
11	Meadow-Pipit	17	Wren
7	Wagtails	19	Hedge-Sparrow
6	Orphean-Warbler	11	Robin
4	Garden-Warbler	12	Yellow-Ammer
3	Red-Start	13	Reed-Warbler
1	Robin	4	Garden-Warbler
1	Sedge-Warbler	6	Meadow-Pipit
1	Barred-Warbler	4	Tree-Pipit
1	Aquatic-Warbler	5	Willow-Warbler
1	Whitethroat	5	Greenfinch
1	Wheatear {sp. Saxicola melanoleuca	4	Chaffinch
1	Pied-Flycatcher	4	Linnet
1	Hedge-Sparrow	3	Pied-Wagtail
Total 39		2	Blackcap
		2	Red-Start
		2	Lesser-Whitethroat
		2	Nightingale
		2	Wood-Warbler
		2	Spotted-Flycatcher
		2	Sedge-Warbler
Fair, or within limits of colour-variation of egg of species		2	Red-backed-Shrike
		1	Woodchat-Shrike
45	Meadow-Pipit	1	Rock-Pipit
19	Wagtails	1	Sky-Lark
11	Tree-Pipit	1	Crested-Lark
7	Reed-Warbler	1	Goldfinch
6	Robin	1	House-Sparrow
5	Blackcap	1	Common-Bunting
4	Whitethroat	1	Cirl-Bunting
2	Marsh-Warbler	1	Great-Reed-Warbler
2	Sedge-Warbler	1	Marsh-Warbler
2	Red-backed-Shrike	1	Grasshopper-Warbler
1	Lesser-Grey-Shrike	1	Dartford-Warbler
1	Lesser-Whitethroat	1	Barred-Warbler
1	Tree-Sparrow	1	Wheatear
1	Common-Bunting	1	Orphean-Warbler
1	Sky-Lark	1	Chiffchaff
1	Crested-Lark	1	Fire-Crested-Wren
Total 109		Total 140	

The appended table of colour-matching gives in a concise form the more important numerical facts. The total number of successes, complete or partial, in colour-matching is so great, nearly 51·4% of the whole series examined, that mere chance is impossible as an explanation. Moreover, the accuracy with which highly remarkable Cuckoo's eggs are deposited in appropriate clutches is so

striking as to tempt one to dally with conscious selection and deliberate choice on the part of the female Cuckoo: for instance, six blue Cuckoo's eggs occur in the series, of these three were in Red Starts' nests (Nos. 155, 158, and one not accessible for measurement), one (No. 159) in nest of *Saxicola melanoleuca*, and two others (not accessible) in nests of Hedge-Sparrow and Pied-Flycatcher. All these birds lay blue eggs, and so far as my observations go, blue Cuckoo's eggs are not deposited elsewhere, though Howard Saunders's statement (*Manual of British Birds*, p. 288) that "these, (sc. blue eggs), have not been invariably located in nests of the Hedge-Sparrow and the Red Start" leads me to suppose that they may have been found in the nests of birds whose eggs are not blue. Again, the egg of the Orphean Warbler is of a very distinct type, and yet in six cases out of seven the Cuckoo's egg deposited in the nest of this species resembles this type to a nicety, nor is this particular variety of Cuckoo's egg to be found in any other nest.

	Matched more or less	Not matched	Total	Percentage matched
Meadow-Pipit ...	56	6	62	90·3%
Wagtails... ...	26	3	29	89·6%
Hedge-Sparrow ...	1	19	20	5·0%
Robin	7	11	18	38·8%
Reed-Warbler ...	7	13	20	35·5%
Tree-Pipit ...	11	4	15	73·3%
Wren	0	17	17	0·0%
Yellow-Ammer ...	0	12	12	0·0%
Garden-Warbler	4	4	8	50·0%
Orphean-Warbler	6	1	7	85·7%

Perhaps the most striking point in connection with colour-matching is its entire absence from the eggs of Wren-Cuckoos, which, though closely resembling one another, in no case match those of the Wren itself. The Wren is the only bird of those dealt with in this paper that constructs a nest of such a character as to render a view of the eggs impossible alike to the Wren and the Cuckoo; hence failure in colour-matching cannot possibly reveal the intruder to the lawful owner and discrepancy in size becomes of more importance. It is very remarkable that, in both length and breadth, as already pointed out, the eggs of Wren-Cuckoos show a far wider variation from the average Cuckoo and in the direction of the Wren's egg than is the case with any other species. It is too a well known fact that the Wren is peculiarly intolerant of interference with her nest—at any rate at the hands of human beings.

To sum up, we note that there are three cases of practically no colour-matching, the Hedge-Sparrow-Cuckoo, the Wren-Cuckoo and the Yellow-Ammer-Cuckoo; in the first two of these cases there is an attempt both as to length and breadth at size-matching. In the third case no significant size-difference is to be found from our data, but these are too scant to be really conclusive.

In spite then of the criticisms expressed above and of the absence of actual

evidence of the ejection of Cuckoo's eggs by small birds, I feel compelled to admit that there is a selective process at work, tending, in many cases, to bring the Cuckoo's eggs into agreement with those of the host both in size and colour, and am inclined to suggest that perhaps there may be *local* "gentes" of Cuckoos which as a rule, but by no means exclusively, patronise the nests of particular species. It seems well established (1) that a Cuckoo returns every year to the same locality, and (2) according to Reh, lays its eggs only in the nests of that particular species which it, or its ancestors, happen to have adopted for that purpose, while the coloration of the egg of every female Cuckoo is peculiar to itself. The evidence that my material furnishes on these points is as follows:—the Cuckoo's eggs in the South Kensington Collection from any one locality frequently exhibit strong resemblances *inter se*, e.g. (*a*) Nos. 107, 108, 109 (Robin-Cuckoo), 180 (Greenfinch-Cuckoo), 189 (Spotted-Flycatcher-Cuckoo), and 61 (Meadow-Pipit-Cuckoo), all taken at Churt in the year 1860, are all so closely similar that they may well be from one and the same bird; (*b*) Nos. 25 and 26 (Meadow-Pipit-Cuckoo) from Lochend, but not dated; (*c*) Nos. 34 and 35 (both in same nest of Meadow-Pipit) from S.W. Lancashire, dated June 29, 1866, and 33 (Meadow-Pipit-Cuckoo) from N.W. Cheshire, dated 25 May, 1866, and all three collected by H. E. Smith; (*d*) No. 193 (Red-backed-Shrike-Cuckoo, June, 1863), 110 (Robin-Cuckoo, June, 1863), 195 (Skylark-Cuckoo, May, 1862), 181 (Greenfinch-Cuckoo, 1864), 243 (Linnet-Cuckoo, June, 1864), and 168 (Yellow-Ammer-Cuckoo, June, 1864), all coming from Churt, and (*e*) Nos. 225—236 (Wren-Cuckoo), Pomerania, 1879, 1880 and 1881, exhibit the same phenomenon in their several sets. But it is to be observed that eggs of the same coloration are not confined to the nests of any one species of foster-parent, except in (*c*) and (*e*). Again, eleven nests contained two Cuckoo's eggs apiece, viz. Nos. 28 and 29, 34 and 35, 36 and 37, 38 and 39, and two others not accessible for measurement, all in Meadow Pipits' nests, 139 and 140 in Orphean Warblers', 210 and 211 in Pied Wagtails', 214 and 215, 217 and 218 in Pied Wagtails', and 145 and 146, 147 and 148 in Reed Warblers', and in each case the two eggs are obviously laid by the same parent. Reh mentions the fact that in 1893 within two kilometres from Leipzig no less than 70 nests were found containing Cuckoo's eggs, and of these, 58 (83 %), were in nests of Red-backed Shrike. It seems possible then that in any given locality a majority of the Cuckoos may favour some one particular species of foster-parent, and if this be so the chances of male and female Cuckoos of like rearing mating together are very largely increased, and the difficulty raised in a preceding passage is to some extent removed. It is however evident that the isolation of "gens" from "gens" is not perfect, and this may perhaps be accounted for by occasional matings between birds of unlike foster-parentage and the offspring inheriting mixed tendencies.

In conclusion I must acknowledge my great indebtedness to Professor Karl Pearson for the interest he has taken in this investigation and for much kind advice and assistance in the statistical portions.

APPENDIX I. *Cuckoo (C. canorus). Length of Egg in millimetres.*

(The names of species refer to the nest in which the eggs were found.)

Clutch Unknown

1. 22·5
2. 20·1
3. 23·3
4. 22·9
5. 23·1
6. 22·0
7. 22·3
8. 23·6
9. 24·7
10. 23·7
11. 24·0
12. 20·4
13. 21·3
14. 22·0
15. 24·2
16. 21·7
17. 21·0
18. 20·1
19. 21·9
20. 21·9

Meadow-Pipit

21. 21·7
22. 22·6
23. 20·9
24. 21·6
25. 22·2
26. 22·5
27. 22·2
28. 24·3
29. 22·3
30. 22·6
31. 20·1
32. 22·0
33. 22·8
34. 22·0
35. 22·4
36. 22·3
37. 20·6
38. 22·1
39. 21·9
40. 23·0
41. 22·0
42. 22·0
43. 22·1
44. 22·0
45. 19·6
46. 22·8
47. 22·0
48. 23·4
49. 23·8
50. 23·3
51. 22·5
52. 22·3
53. 21·9
54. 22·0
55. 21·7
56. 23·3
57. 22·2
58. 22·3
59. 22·8
60. 22·9
61. 23·7
62. 22·0
63. 21·9
64. 22·2
65. 24·4

Tree-Pipit

66. 22·7
67. 23·3
68. 24·0
69. 23·6
70. 22·1
71. 21·8
72. 21·1
73. 23·4
74. 23·8
75. 23·3
76. 24·0
77. 23·5
78. 23·2
79. 24·0
80. 22·4

Rock-Pipit

81. 23·9

Hedge-Sparrow

82. 22·0
83. 23·9
84. 20·9
85. 23·8
86. 25·0
87. 24·0
88. 21·7
89. 23·8
90. 22·8
91. 23·1
92. 23·1
93. 23·5
94. 23·0
95. 23·0

Robin

96. 21·8
97. 23·0
98. 23·3
99. 22·4
100. 22·4
101. 23·0
102. 23·0
103. 23·0
104. 23·9
105. 22·3
106. 22·0
107. 22·6
108. 22·0
109. 22·1
110. 21·1
111. 23·0

Blackcap

112. 21·3
113. 19·9
114. 22·9
115. 23·3
116. 22·1
117. 20·9

Whitethroat

118. 21·9
119. 22·9
120. 22·4
121. 19·1

Chiffchaff

122. 20·9

Sedge-Warbler

123. 23·0
124. 21·0
125. 22·0
126. 21·3

Wood-Warbler

127. 22·0
128. 22·0

Willow-Warbler

129. 21·3
130. 21·1
131. 23·1
132. 21·0

Garden-Warbler

133. 23·0
134. 23·1
135. 22·9
136. 23·2

Orphean-Warbler

137. 24·4
138. 21·5
139. 23·3
140. 22·3

Reed-Warbler

141. 23·2
142. 22·0
143. 22·2
144. 21·2
145. 21·6
146. 21·6
147. 21·9
148. 22·0
149. 22·9
150. 22·8

Marsh-Warbler

151. 22·7

Lesser-Whitethroat

152. 21·0
153. 22·5

Barred-Warbler

154. 21·9

Red-Start

155. 24·0
156. 23·2
157. 22·3
158. 23·0

Saxicola-Melanoleuca

159. 23·1

Reed-Bunting

160. 23·2
161. 20·9

Yellow-Ammer

162. 22·5
163. 21·9
164. 22·5
165. 23·3
166. 23·2
167. 23·0
168. 21·1
169. 22·9
170. 23·3

Corn-Bunting

171. 20·1
172. 22·9

Chaffinch

173. 23·3
174. 22·3
175. 22·9
176. 21·2

Goldfinch

177. 20·9

Greenfinch

178. 22·7
179. 22·8
180. 22·1
181. 23·4
182. 21·2
183. 22·5

House-Sparrow

184. 23·9

Tree-Sparrow

185. 24·0

Nightingale

186. 22·8
187. 23·2

Spotted-Flycatcher

188. 22·1
189. 22·4

Lesser-Grey-Shrike

190. 23·0

Woodchat-Shrike

191. 22·0

Red-backed-Shrike

192. 22·1
193. 20·8

Sky-Lark

194. 22·2
195. 21·2

Crested-Lark

196. 22·5
197. 21·1

Pied-Wagtail

198. 23·0
199. 23·4
200. 24·0
201. 23·3
202. 23·1
203. 22·4
204. 21·8
205. 21·8
206. 24·9
207. 24·0
208. 22·1
209. 21·0
210. 22·6
211. 21·9
212. 24·0

White-Wagtail

213. 22·3
214. 22·6
215. 22·0
216. 22·7
217. 22·3
218. 22·5

Blueheaded-Yellow-Wagtail

219. 21·2
220. 22·4

Yellow-Wagtail

221. 22·2
222. 22·2
223. 23·0

Wren

224. 19·8
225. 22·1
226. 21·5
227. 20·9
228. 22·0
229. 21·0
230. 22·3
231. 21·0
232. 20·3
233. 20·9
234. 22·0
235. 20·0
236. 20·8
237. 21·2
238. 21·0

Fire-Crested-Wren

239. 24·2

Wheatear

240. 22·8

Linnet

241. 24·7
242. 24·0
243. 22·9

APPENDIX II. *Cuckoo* (*C. canorus*). *Breadth of Egg in millimetres.*

Clutch Unknown

1. 17·0
2. 14·9
3. 16·0
4. 17·4
5. 17·4
6. 16·5
7. 17·2
8. 17·2
9. 18·0
10. 17·8
11. 18·0
12. 15·0
13. 16·0
14. 16·5
15. 17·3
16. 16·9
17. 16·1
18. 15·8
19. 15·9
20. 16·2

Meadow-Pipit

21. 16·1
22. 17·0
23. 16·2
24. 16·2
25. 16·9
26. 16·9
27. 17·3
28. 16·8
29. 16·8
30. 17·0
31. 16·5
32. 16·9
33. 16·5
34. 17·0
35. 17·0
36. 16·3
37. 16·2
38. 16·8
39. 17·0
40. 16·9
41. 17·0
42. 17·0
43. 17·3
44. 16·8
45. 15·8
46. 17·1
47. 16·9
48. 16·4
49. 16·4
50. 16·8
51. 17·1
52. 17·0
53. 17·1
54. 17·2
55. 16·2
56. 16·7
57. 16·8
58. 16·2
59. 16·4
60. 17·2
61. 17·0
62. 17·2
63. 17·0
64. 16·2
65. 16·2

Tree-Pipit

66. 16·3
67. 16·6
68. 17·0
69. 16·9
70. 16·3
71. 16·7
72. 16·5
73. 16·2
74. 16·3
75. 16·7
76. 17·5
77. 17·3
78. 16·4
79. 17·3
80. 16·0

Rock-Pipit

81. 16·4

Hedge-Sparrow

82. 17·0
83. 16·9
84. 15·8
85. 17·3
86. 17·5
87. 17·5
88. 16·2
89. 16·5
90. 16·2
91. 17·1
92. 16·1
93. 16·9
94. 16·7
95. 17·0

Robin

96. 16·0
97. 15·9
98. 17·1
99. 16·6
100. 16·9
101. 16·1
102. 17·2
103. 16·2
104. 16·9
105. 15·2
106. 16·3
107. 17·0
108. 16·0
109. 16·4
110. 16·4
111. 17·0

Blackcap

112. 16·1
113. 16·0
114. 16·0
115. 16·1
116. 17·8
117. 15·3

Whitethroat

118. 17·0
119. 16·4
120. 16·6
121. 14·0

Chiffchaff

122. 15·3

Sedge-Warbler

123. 16·0
124. 16·1
125. 16·3
126. 16·5

Wood-Warbler

127. 16·2
128. 15·2

Willow-Warbler

129. 15·8
130. 15·8
131. 16·6
132. 16·0

Garden-Warbler

133. 16·2
134. 16·8
135. 16·0
136. 17·0

Orphean-Warbler

137. 17·9
138. 16·0
139. 16·4
140. 16·3

Reed-Warbler

141. 16·9
142. 17·1
143. 17·0
144. 16·1
145. 16·5
146. 16·5
147. 16·1
148. 16·5
149. 17·9
150. 16·5

Marsh-Warbler

151. 16·7

Lesser-Whitethroat

152. 16·0
153. 16·2

Barred-Warbler

154. 16·8

Red-Start

155. 17·7
156. 16·2
157. 16·7
158. 17·0

Saxicola-Melanoleuca

159. 17·1

Reed-Bunting

160. 16·9
161. 15·9

Yellow-Ammer

162. 16·0
163. 16·0
164. 16·1
165. 17·2
166. 16·5
167. 17·0
168. 17·0
169. 17·0
170. 16·8

Corn-Bunting

171. 15·8
172. 17·0

Chaffinch

173. 16·1
174. 16·2
175. 17·3
176. 15·7

Goldfinch

177. 16·0

Greenfinch

178. 14·5
179. 16·7
180. 16·9
181. 17·0
182. 16·2
183. 17·0

House-Sparrow

184. 17·7

Tree-Sparrow

185. 16·0

Nightingale

186. 17·2
187. 16·0

Spotted-Flycatcher

188. 15·8
189. 16·5

Lesser-Grey-Shrike

190. 16·8

Woodchat-Shrike

191. 17·0

Red-backed-Shrike

192. 17·1
193. 15·7

Sky-Lark

194. 18·1
195. 15·6

Crested-Lark

196. 16·4
197. 15·9

Pied-Wagtail

198. 16·3
199. 16·7
200. 17·0
201. 16·3
202. 16·7
203. 16·5
204. 16·0
205. 16·0
206. 16·8
207. 15·8
208. 16·2
209. 17·1
210. 16·0
211. 16·9
212. 17·2

White-Wagtail

213. 16·8
214. 17·0
215. 17·0
216. 16·9
217. 17·3
218. 16·9

Blueheaded-Yellow-Wagtail

219. 15·9
220. 17·2

Yellow-Wagtail

221. 16·7
222. 16·4
223. 16·3

Wren

224. 15·0
225. 16·0
226. 16·2
227. 15·7
228. 16·2
229. 15·5
230. 16·0
231. 15·9
232. 15·5
233. 15·9
234. 16·0
235. 15·7
236. 15·9
237. 16·0
238. 16·0

Fire-Crested-Wren

239. 16·9

Wheatear

240. 16·6

Linnet

241. 16·3
242. 18·8
243. 17·1

APPENDIX III.

Frequency Distribution of Cuckoo's Eggs.

Length	Number	Breadth	Number
18·75—19·25	1	13·75—14·25	1
19·25—19·75	1	14·25—14·75	1
19·75—20·25	7	14·75—15·25	5
20·25—20·75	3	15·25—15·75	9
20·75—21·25	29	15·75—16·25	73
21·25—21·75	13	16·25—16·75	51
21·75—22·25	54	16·75—17·25	80
22·25—22·75	38	17·25—17·75	15
22·75—23·25	47	17·75—18·25	7
23·25—23·75	22	18·25—18·75	0
23·75—24·25	21	18·75—19·25	1
24·25—24·75	5		
24·75—25·25	2		
	243		243

These distributions are fitted with normal curves in the accompanying diagrams.

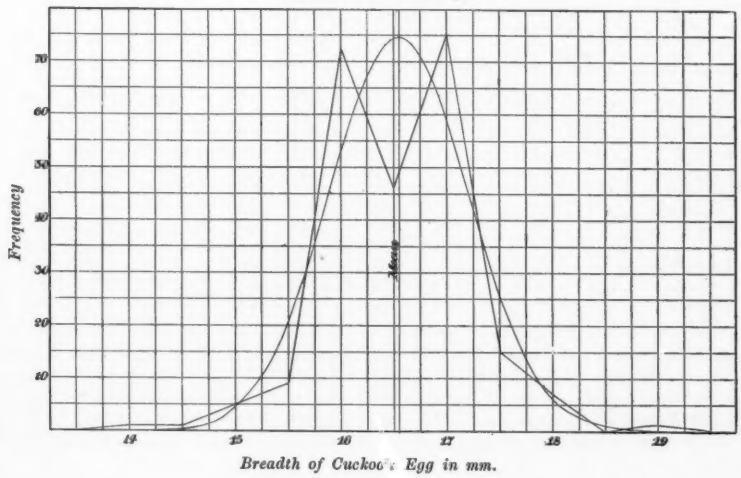
Length $y = 45{\cdot}793\, e^{-\frac{1}{2}x^2/(2{\cdot}117)^2}$.

Origin at 22·40.

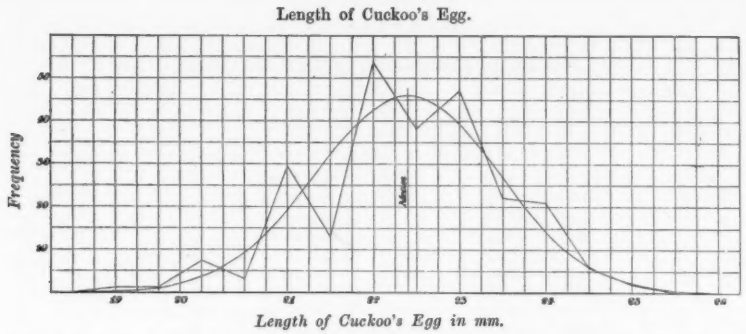
Breadth $y = 74{\cdot}618\, e^{-\frac{1}{2}x^2/(1{\cdot}2992)^2}$.

Origin at 16·54.

Breadth of Cuckoo's Egg.



Breadth of Cuckoo's Egg in mm.



The curves give fairly reasonable graduated values, considering: (i) the paucity of data, and (ii) the possibility of class differences within the race indicated in this memoir.

APPENDIX IV.

Table of Egg Measurements.

The following is a summary of my measurements on the Cuckoo's and other birds' eggs. I have added the results of recent measurements on the eggs of House-Sparrow, Blackbird, Song-Thrush, Starling and Linnet made by Professor Pearson and some of his co-workers. The whole serves to illustrate the relative smallness of the Cuckoo's egg.

Bird	Approximate length in inches	No. of Cases	Length of Egg			Breadth of Egg		
			Mean	S. D.	C. of V.	Mean	S. D.	C. of V.
Cuckoo ...	14	243	22·40	1·0585	4·72	16·54	·6496	3·93
Blackbird ...	10	114	29·44	1·3568	4·61	21·73	·7874	3·62
Song-Thrush ...	9	151	27·44	·9988	3·64	20·69	·5162	2·50
Starling ...	8—8·5	27	29·78	1·0973	3·68	21·76	·4233	1·94
Wagtail* ...	7—8	16	20·75	1·4448	6·96	14·67	·3703	2·52
Yellow-Ammer	7	32	21·55	·6821	3·17	16·04	·4045	2·53
Tree-Pipit ...	6·5	27	20·01	·6978	3·49	15·09	·4488	2·97
Meadow-Pipit	6	74	19·72	1·2504	6·37	14·56	·5611	3·84
House-Sparrow	6	687	21·82	1·1946	5·47	15·51	·5245	3·38
Hedge-Sparrow	6	26	20·12	·8096	4·02	14·73	·4146	2·81
Robin	6	57	20·22	·8565	4·24	15·43	·4771	3·09
Linnet... ...	5·5—6	65	17·14	·5984	3·49	13·33	·3581	2·69

* This was a mixed series made up of 6 eggs of White-Wagtail, 7 of Pied-Wagtail and 3 of Blue-headed-Yellow-Wagtail. This accounts for the great variability in length. We see that the Cuckoo's egg is the most variable of the whole series in breadth and with the exception of the Meadow-Pipit's and House-Sparrow's the most variable also in length. The biggest of all the birds here dealt with, the Cuckoo has an egg hardly longer than the House-Sparrow's or broader than the Yellow-Ammer's.

ON CRIMINAL ANTHROPOMETRY AND THE IDENTIFICATION OF CRIMINALS.

By W. R. MACDONELL, M.A., LL.D.

[Received November 5, 1901.]

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PART I.

Material and Methods.

(1) The object of this memoir is threefold:

(i) To test to what extent the criminal classes diverge in physical characters from other classes of the community.

(ii) To consider how far the shorter methods recently proposed by Professor Karl Pearson for finding the variability and correlation of characters in the case of normal frequency may be applied to some of the chief anthropometric measurements now customarily made, and

(iii) To determine what is the best manner in which these measurements can be applied to the identification of criminals.

I shall first consider the material I have had at my disposal; I shall then indicate the methods I have used for the determination of its metrical constants, and finally apply my results to the consideration of the above three problems. In the course of my work I shall have to consider the important point of the homogeneity and normality of my material, and apply three separate tests:

(i) If broken up into groups, the statistical constants of a sub-group ought to remain, within the limits of random sampling, the same as the bulk of the observations.

(ii) Frequency distributions, if plotted and fitted with frequency curves, ought to give nearly normal distributions.

(iii) The regression lines in type cases ought to be closely represented by straight lines.

(2) *Nature of material used.* The data on which the memoir is based were obtained, through the kindness of Dr Garson, from the Central Metric Office, New Scotland Yard, where the register of habitual criminals is kept, and their identification effected. In an interesting paper published in the *Journal of the Anthropological Institute*, Vol. xxx. 1900, July—Dec., Dr Garson has explained in detail the metric system of identification in force in England, and the sort of prisoners whose metric description is registered at the Central Office, and whom we may call briefly "habitual" criminals.

But in addition to the official forms on which the descriptions of habitual criminals are recorded, the Central Office possesses a number of "practice" forms which record the metric description of a less pronounced type of criminal than the "habituals"—prisoners whose crimes and sentences are comparatively slight, and who may be called "non-habitual." These practice forms are filled up by warders

on probation, and as their advancement in the Service depends on the accuracy with which they measure and describe the prisoners, they may be expected to do so with the utmost care. Indeed, I understand from Dr Garson that these "practice" forms may be accepted with as much confidence as the official forms for habitual criminals.

It is with these non-habitual criminals that we are concerned. Obviously, the larger the number of individuals we include, the better will be the results; on the other hand, the number must not be extremely large, otherwise the investigation becomes too laborious. Accordingly a group of 3000 was decided on as likely to give good results, while still keeping the work within manageable compass. Three thousand metric forms were thereupon obtained from Scotland Yard, containing particulars of 3000 male prisoners undergoing their sentences in the chief prisons of England and Wales. The majority of the prisoners were English and Welsh, many were Irish, and only a few Scotch; no foreigners or youths under 21 were included. The forms were drawn at random from the mass on the office shelves; we are therefore dealing with a random sampling.

The metric description recorded on the forms includes (*inter alia*) certain physical measurements, viz., those of the Head Length, Head Breadth, Face Breadth, Left Middle Finger, Left Cubit, Left Foot, and Height, of each individual. These measurements are taken, in the case of height, to the nearest $\frac{1}{8}$ of an inch, in the case of the other characters, to the nearest millimetre; thus, Head Length of 19·2 centimetres includes all head lengths from 19·15 cm. to 19·25 cm.; Height of 5 ft. $5\frac{1}{8}$ in. includes all heights from 5 ft. $5\frac{11}{16}$ in. to 5 ft. $5\frac{3}{16}$ in. These characters form the subject of the present memoir, and will be studied from the anthropometrical point of view in the earlier part of the paper; the problem of identification, with which Scotland Yard is concerned, will be discussed in the latter part.

(3) *Methods employed.* In dealing with this large mass of observations, I propose to apply the methods explained in Professor Karl Pearson's memoirs in the *Philosophical Transactions*, Vol. 195, A., pp. 1—47, 79—150, in order to calculate the Means, Standard Deviations, and Coefficients of Correlation, with their probable errors, for the above seven characters. These methods were devised by Professor Pearson to deal with characters not quantitatively measureable, but they are used here with the object of saving much of the labour involved in making these calculations in the ordinary way from the usual elaborate correlation tables. It will be seen later on that the saving of time and labour is very considerable.

To obtain the coefficients of correlation Professor Pearson shows (*loc. cit.* p. 2) that a 4-fold table is required; to find standard deviations and means, and probable errors and error correlations of all the quantities involved, a 9-fold table is required (*loc. cit.* pp. 82, 84). If a 16-fold table is formed, it will give the required 4-fold and 9-fold tables, as is shown in the following example, which is a frequency correlation table for the Head Length and Face Breadth of the 3000 criminals.

Head Length (in cms.)

Face Breadth (in cms.) Class	*18·8* and under	*18·9* to *19·1*	*19·2* to *19·3*	*19·4* and over	Totals
13·4 and under	453	222	135	229	1039
13·5 to *13·6* ...	147	99	63	152	461
13·7 to *13·8* ...	112	99	74	176	461
13·9 and over	146	177	156	560	1039
Totals	858	597	428	1117	3000

From this the following 9-fold table is constructed:

Head Length.

Face Breadth. Class	*18·8* and under	*18·9* to *19·3*	*19·4* and over	Totals
13·4 and under	453	357	229	1039
13·5 to *13·8* ...	259	335	328	922
13·9 and over	146	333	560	1039
Totals	858	1025	1117	3000

and finally a 4-fold table to find coefficient of correlation:

Head Length.

Face Breadth. Class	*19·1* and under	*19·2* and over	Totals
13·6 and under	921	579	1500
13·7 and over	534	966	1500
Totals	1455	1545	3000

(It may be remarked that this latter table would also give the means, but we shall use the 9-fold table for this purpose.)

It seemed at the outset that the best results would be obtained by choosing the limits of divisions so that the totals of each column and of each row in the 4-fold and 9-fold tables should be as nearly as possible equal to a half and a third respectively of the whole 3000; but this principle was found in some cases to be inapplicable*.

(4) Before making use of these short tables we will first consider the results obtained in the case of our 3000 criminals from three frequency correlation tables (Tables I. II. and III.) made up in the usual way. The methods of preparing the tables and calculating the results, with their probable errors, are very fully explained

* In fixing the limits much help was obtained from Dr Garson's results.

in Mr G. Udny Yule's paper "On the Theory of Correlation" in the *Journal of the Royal Statistical Society*, Vol. LX., Part IV., December 1897. We have modified Mr Yule's formula for the square of the standard deviation by diminishing it by the quantity $\frac{1}{12}$, in accordance with Mr W. F. Sheppard's paper in the *Proceedings of the London Mathematical Society*, Vol. XXIX., Nos. 634/5.

The results are as follows:

TABLE 1.

3000 *Criminals.*

	Standard Deviation	Mean
Head Length (cm.) ...	·6046 ± ·0053	19·1663 ± ·0075
Head Breadth (cm.)...	·5014 ± ·0044	15·0442 ± ·0062
L. M. Finger (cm.) ...	·5479 ± ·0048	11·5474 ± ·0068
Height (ins.)	2·5410 ± ·0221	65·5355 ± ·0313

TABLE 2.

3000 *Criminals.*

	Coefficient of Correlation
Head Length and Head Breadth	·4016 ± ·0103
Head Breadth and Height ...	·1831 ± ·0119
L. M. Finger and Height ...	·6608 ± ·0069

(5) *Tests of Normality.* At the foot of Tables II. and III. (see pp. 215, 216) are shown the mean Finger Length and mean Head Breadth of each column array; these means are plotted on Figs. 1 and 2, and the lines are drawn which show the theoretical regression of Finger on Height, and Head Breadth on Height. The slope of these lines is calculated from the formula $\tan\theta = \frac{r\sigma_2}{\sigma_1}$, where r is the coefficient of correlation, and σ_1, σ_2, the standard deviations of the correlated organs. (See Yule, *loc. cit.*) The means of the column arrays at the extremities of the tables are not included, as they are based on such small frequencies.

It will be observed that the regression lines fit the observations very well, i.e., the regression is very closely linear.

FIG. 1. Line of Regression: Left Middle Finger on Height. 3000 Criminals.
Mean Height 65·5. Mean Finger 115·5. Tan $\theta = 1{\cdot}425$.

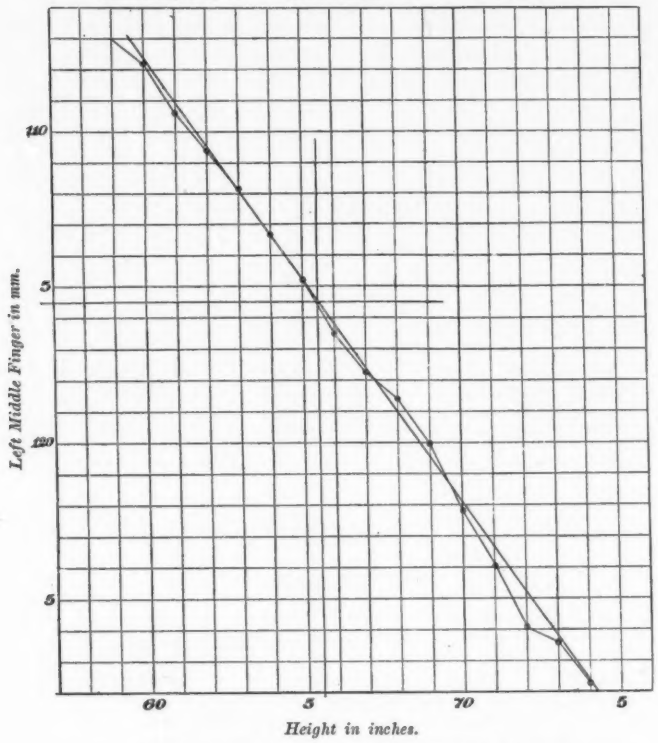
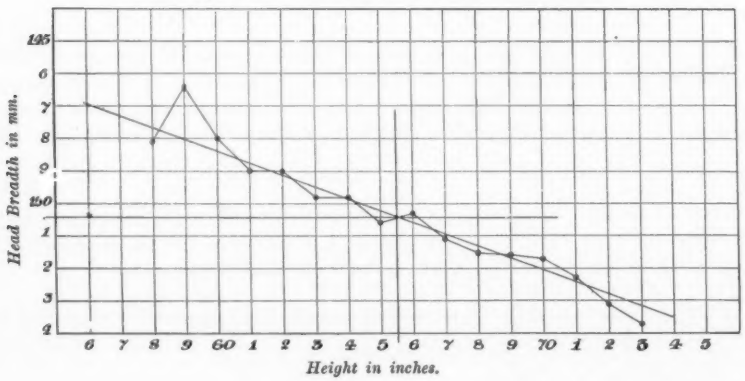


FIG. 2. Line of Regression: Head Breadth on Height. 3000 Criminals.
Mean Height 65·5. Mean Breadth 150·4. Tan $\theta = {\cdot}3612$.



We will next attempt to fit a curve to the observed frequencies of Head Breadth and Height in the case of the 3000 criminals in accordance with Professor Pearson's method (see *Phil. Trans.* Vol. 186, A. pp. 343—414).

(i) Head Breadth.

The moments are:

$$\mu_2 = 25{\cdot}14526 \qquad \beta_1 = {\cdot}010385$$
$$\mu_3 = 12{\cdot}84962 \qquad \beta_2 = 3{\cdot}032609$$
$$\mu_4 = 1917{\cdot}47151.$$

Therefore the criterion $2\beta_2 - 3\beta_1 - 6 = +{\cdot}03406$; the curve is therefore of Type IV., and the remaining constants are:

$$d = -\ {\cdot}253365 \qquad \text{Skewness} = -\ {\cdot}050526$$
$$r = 356{\cdot}203035 \qquad m = 179{\cdot}101518$$
$$\nu = -194{\cdot}9783 \qquad a = 82{\cdot}900557$$
$$\log y_0 = \overline{20}{\cdot}3543484.$$

The equation to the curve is:

$$x = 82{\cdot}900557 \tan\theta,$$
$$\log y = \overline{20}{\cdot}3543484 + 358{\cdot}203035 \log\cos\theta + 1{\cdot}477904\,\theta.$$

The maximum ordinate or mode is 239·39 and occurs at $x = 45{\cdot}124714$, corresponding to a head breadth of 15·0188; the mean or centroid occurs at $x = 45{\cdot}378079$, its value being 15·0442.

The curve obviously approximates very closely to the normal curve.

(ii) Height.

$$\mu_2 = 6{\cdot}45660 \qquad \beta_1 = {\cdot}002578$$
$$\mu_3 = {\cdot}83300 \qquad \beta_2 = 3{\cdot}175076$$
$$\mu_4 = 132{\cdot}36172.$$
$$2\beta_2 - 3\beta_1 - 6 = +{\cdot}342418.$$

The curve is again of Type IV., and the remaining constants are:

$$d = -\ {\cdot}058071 \qquad \text{Skewness} = -\ {\cdot}022853$$
$$r = 38{\cdot}067473 \qquad m = 20{\cdot}033736$$
$$\nu = -2{\cdot}870793 \qquad a = 15{\cdot}42649$$
$$y_0 = 433{\cdot}647.$$

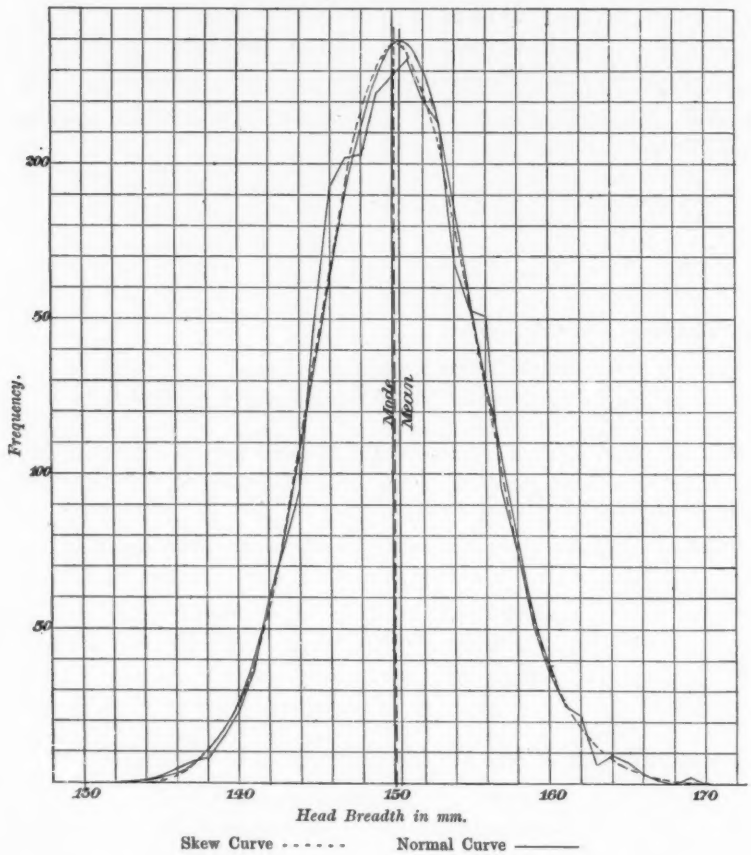
The equation to the curve is:

$$x = 15{\cdot}42649 \tan\theta,$$
$$y = 433{\cdot}647\ (\cos\theta)^{40{\cdot}067473}\ e^{2{\cdot}870793\,\theta}.$$

The mode is 480·574 and occurs at $x = 1{\cdot}105291$, corresponding to a height of 65″·4774; the mean occurs at $x = 1{\cdot}163362$, its value being 65″·5355.

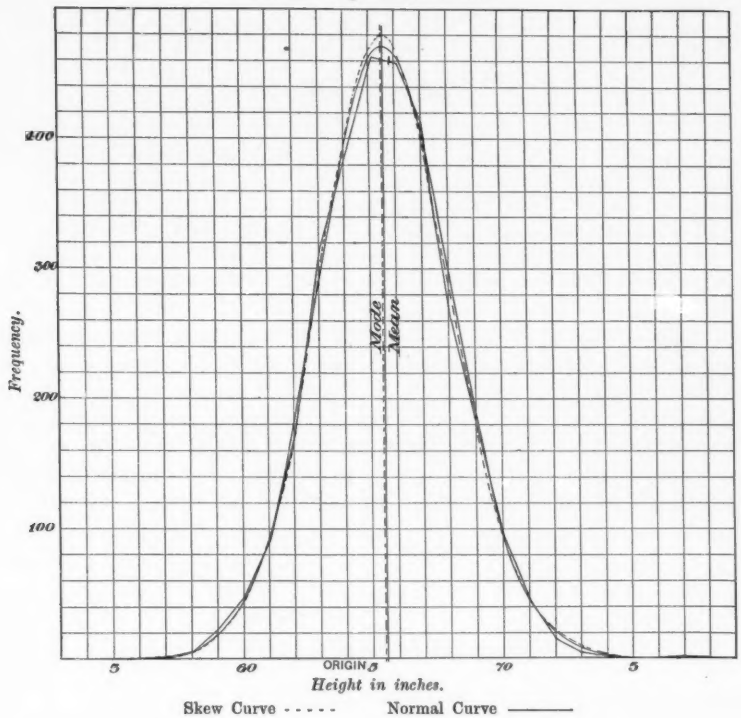
This curve also is very nearly normal. Both curves are drawn on Figs. 3 and 4, which bring out clearly the closeness of the two curves to each other.

FIG. 3. Head Breadth in Criminals.



Seeing that the regression lines are closely linear, and the frequency curves closely normal, we may assert that the distribution is at any rate very approximately normal. This is an important result, as it gives us confidence in applying Professor Pearson's abbreviated method, based as it is on the assumption of normal frequency.

FIG. 4. Height of Criminals.



Skew Curve - - - - - Normal Curve ———

(6) *Tests of Homogeneity.* In order to see whether or not we are dealing with a homogeneous class, three frequency correlation tables (Tables IV. V. and VI.; see pp. 217—219) were formed from the first batch of forms taken at random from Scotland Yard, and comprising particulars of 1306 out of our 3000 criminals. These tables give the following results, the method of calculation being the same as before:

TABLE 3.

1306 *Criminals.*

	Standard Deviation	Mean
Head Length (cm.) ...	·6002 ± ·0079	19·1438 ± ·0112
Head Breadth (cm.) ...	·5133 ± ·0068	15·0302 ± ·0096
L. M. Finger (cm.) ...	·5451 ± ·0072	11·5244 ± ·0102
Height (ins.)	2·5906 ± ·0342	65·4193 ± ·0407
Face Breadth (cm.) ...	·5022 ± ·0066	13·6350 ± ·0094

TABLE 4.

1306 *Criminals.*

	Coefficient of Correlation
Head Length and Head Breadth	·3958 ± ·0157
Head Breadth and Height ...	·1973 ± ·0179
L. M. Finger and Height ...	·6682 ± ·0103

To compare the results thus obtained for the two sets of 3000 and 1306, we will compare the absolute difference of their standard deviations, means, and coefficients of correlation with the square root of the sums of the squares of the probable errors. Let us call the difference D, the square root in question S, the coefficient of correlation r; and to show the form of the calculation we will take the standard deviation of Head Length as an example. For the sake of keeping as many significant figures as possible, we will use the millimetre as our unit.

S.D. of 1306 criminals is 6·00247 ± ·07922

„ 3000 „ 6·04593 ± ·05265

$D = \cdot 04346$

$\{(\cdot 07922)^2 + (\cdot 05265)^2\}^{\frac{1}{2}} = S = \cdot 09512$

Thus it appears that D is less than one-half of S.

Collecting all our results together, we have the following tables:

TABLE 5.

1306 *and* 3000 *Criminals.*

	Standard Deviation		Mean	
	D	*S*	*D*	*S*
Head Length	·04346	·09512	·22535	·13451
Head Breadth ...	·11840	·08059	·13922	·11397
Height...	·04963	·04072	·11618	·05759
L. M. Finger	·02758	·08632	·22941	·12208

TABLE 6.

1306 *and* 3000 *Criminals.*

	Coefficient of Correlation	
	D	*S*
Head Length and Head Breadth	·00586	·01884
Head Breadth and Height ...	·01420	·02154
Finger and Height	·00735	·01245

From these figures it is apparent that there is no sensible difference between the 1306 and the 3000; *D* is not in any case, *even in the means*, twice to thrice *S*. We may therefore treat our material for practical purposes as *normal* and *homogeneous*.

Before leaving this group of 1306, I may add that having formed correlation tables for Head Length and Left Middle Finger, Head Length and Face Breadth, I find the coefficients of correlation to be as follows:

TABLE 7.

1306 *Criminals.*

	Coefficient of Correlation
Head Length and Finger ...	·2861 ± ·0171
Head Length and Face Breadth	·4074 ± ·0156

(7) *Comparison of classes from which criminals are drawn with the educated classes of the community.* It will now be interesting to compare our 3000 criminals with an entirely different social class of the population, and for this purpose I have prepared frequency correlation Tables VII., VIII. and IX. showing the correlation between Head Length and Head Breadth, Height and Head Breadth, Height and Head Length, in the case of 1000 male students at Cambridge. The figures are taken from cards in the possession of the Cambridge Anthropometrical Committee*, and are given in inches, but for Head Length and Head Breadth I have reduced the results to centimetres in order to facilitate comparison.

* Thanks are due to the Committee for their courtesy in allowing the use of their cards.

TABLE 8.

1000 *Cambridge Men.*

	S. D.	Mean
Head Length (cms.) ...	·6161 ± ·0093	19·3509 ± ·0131
Head Breadth (cms.)...	·5055 ± ·0076	15·3959 ± ·0108
Height (ins.)	2·5447 ± ·0384	68·8550 ± ·0543

TABLE 9.

1000 *Cambridge Men.*

	Coefficient of Correlation
Head Length and Head Breadth ...	·3448 ± ·0188
Height and Head Breadth	·1529 ± ·0208
Height and Head Length	·2816 ± ·0196

We will now calculate D and S for the 3000 criminals and 1000 Cambridge men:

TABLE 10.

1000 *Cambridge Men and* 3000 *Criminals.*

	S. D.		Mean	
	D	*S*	*D*	*S*
Head Length ...	·0115	·0107	·1846	·0151
Head Breadth ...	·0041	·0088	·3518	·0124
Height	·0037	·0443	3·3195	·0626

TABLE 11.

1000 *Cambridge Men and* 3000 *Criminals.*

	Coefficient of Correlation	
	D	*S*
Head Length and Head Breadth ...	·0568	·0214
Height and Head Breadth	·0301	·0240

We will next give the coefficient of variation, which is defined as the percentage variation in the mean, the standard deviation being treated as the total variation in the mean (*Phil. Trans.* Vol. 187, A., p. 277).

TABLE 12.

Coefficients of Variation. 1000 *Cambridge Men and* 3000 *Criminals.*

	1000 Cambridge Men	3000 Criminals
Head Length ...	3·1839 ± ·0481	3·1544 ± ·0275
Head Breadth ...	3·2836 ± ·0496	3·3332 ± ·0291
Height	3·6958 ± ·0558	3·8773 ± ·0338

The probable error of the coefficient of variation, v, is calculated from the formula:

$$\text{probable error} = \cdot 6745 \frac{v}{\sqrt{2n}} \left[1 + 2\left(\frac{v}{100}\right)^2\right]^{\frac{1}{2}},$$

where n is the number of measurements. (See Pearson, *Proceedings of the Royal Society*, Vol. 61, p. 345.)

From an examination of these results it appears that there is but little difference in variability or correlation between the criminal and the educated classes, but a most noteworthy difference in *means*. It is also to be noted that there is practically no difference in variability whether measured absolutely or by coefficients of variation.

Finally, we will compare the cephalic index in Cambridge men and criminals. In calculating the index it is to be noted (i) that 11 millimetres are deducted from the head measurements, in accordance with Dr Alice Lee's memoir "A First Study of the Correlation of the Human Skull" (*Phil. Trans.* Vol. 196, A., p. 252); and (ii) that we know only the mean Head Breadth and mean Head Length, and therefore in calculating the mean of the Cephalic Index, which is 100 times the ratio of Head Breadth and Head Length, we must apply the formulæ given in Professor Pearson's paper in the *Proceedings of the Royal Society*, Vol. 60, p. 492. He there shows that if x_1, x_2, be the absolute sizes of any two correlated organs, m_1, m_2, their means, σ_1, σ_2, their standard deviations, r_{12} their coefficient of correlation, i_{12} the mean value of $\frac{x_1}{x_2}$, Σ_{12} the standard deviation of $\frac{x_1}{x_2}$, then

$$i_{12} = \frac{m_1}{m_2}(1 + v_2^2 - r_{12}v_1v_2),$$

$$\Sigma_{12} = i_{12}(v_1^2 + v_2^2 - 2r_{12}v_1v_2)^{\frac{1}{2}},$$

where $v_1 = \sigma_1/m_1, \quad v_2 = \sigma_2/m_2.$

Applying these formulæ we find:

TABLE 13.

Cephalic Index. 1000 *Cambridge Men and* 3000 *Criminals.*

	S. D.		Mean	
	Head Index	Skull Index	Head Index	Skull Index
1000 Cambridge Men ...	2·9478	2·9021	79·562	78·330
3000 Criminals	2·7900	2·7435	78·538	77·227

The great difference in cephalic index between the two classes is to be noted, and it is interesting to observe that there is a large difference in the skull capacity as measured by Dr Lee's formula (*Phil. Trans.* Vol. 196, A., p. 235),

$$C = 6{\cdot}752\,L + 11{\cdot}421\,B - 1434{\cdot}06,$$

where C = capacity in cubic centims, L and B the length and breadth of skull in millims.

For Criminals $L = 191{\cdot}663 - 11 = 180{\cdot}663$
$B = 150{\cdot}442 - 11 = 139{\cdot}442.$

For Cambridge Men $L = 193{\cdot}509 - 11 = 182{\cdot}509$
$B = 153{\cdot}959 - 11 = 142{\cdot}959.$

Calculating C by the formula, we find it is

For Criminals 1378·34
For Cambridge Men 1430·98

but in the absence of the measurement of the height of the skull, we do not lay much stress on these determinations.

Summing up the results of this part of the inquiry, I conclude that there is a substantial difference in stature, and in size and shape of head between the two classes; I do not assert that the source of the criminality is to be found in this difference, but only that criminals are drawn from a different section of the community. As bearing on this point it is worth noting that the mean height in Galton's middle-class measurements at the International Exhibition of 1884, viz. 67″·9, approaches our criminal mean more closely than does the Cambridge mean.

I should add that I believe the head measurements in the prisons and at Cambridge are made practically in the same way.

For further comparison I give the general position as to stature and cephalic index of our criminal classes.

TABLE 14.

Stature (Men).

	Number	Mean, cms.	Standard Deviation, cms.	Coefficient of Variation
Cambridge Students...	1000	174·88	6·46	3·70
English Sons* ...	1078	174·40	6·94	3·98
English Fathers* ...	1078	171·95	6·87	3·99
U.S.A. Recruits† ...	25878	170·94	6·56	3·84
N.S. Wales Criminals‡	2862	169·88	6·58	3·80
Frenchmen§	284	166·80	6·47	3·88
English Criminals ...	3000	166·46	6·45	3·88
Germans§	390	165·93	6·68	4·02

* Calculated from family measurement data, collected by Professor Pearson.

† Pearson: *Phil. Trans.*, Vol. 186, A., p. 386.

‡ Powys: *Biometrika*, Part 1. Vol. 1. p. 44.

§ Pearson: *The Chances of Death*, Vol. 1. p. 295.

TABLE 15.

Cephalic Index.

	Number	Mean	Standard Deviation
Bavarian Peasants* ...	100	83·41	3·58
Baden Recruits* ...	6748	81·15	3·63
French Peasants* ...	56	79·79	3·84
Cambridge Students ...	1000	78·33	2·90
Criminals	3000	77·23	2·74
Brahmans of Bengal* ...	100	76·86	3·65
Mahomedans of Bengal*	100	75·77	3·37
Whitechapel English* ...	107	74·73	3·31

* Professor Pearson's *The Chances of Death*, Vol. 1. p. 290. As his results are based either on skull measurements or on "corrected" head measurements, I have used the "corrected" figures of Table 13.

PART II.

Application of Methods.

(8) I will now proceed to apply the abbreviated method already described to the calculation of coefficients of correlation, standard deviations and means, and I will begin with Height and Head Breadth.

First, the following 16-fold table was formed (see Table II., p. 215).

3000 *Criminals.*

Height (feet and inches).

Head Breadth (cms.)		$5\cdot4\frac{9}{16}$ and under	$5\cdot4\frac{9}{16}$—$5\cdot5\frac{9}{16}$	$5\cdot5\frac{9}{16}$—$5\cdot6\frac{9}{16}$	over $5\cdot6\frac{9}{16}$	Totals
	14·8 and under	455	163	160	299	1077
	14·9—15 ...	158	71	71	151	451
	15·1—15·2 ...	158	70	65	162	455
	over *15·2* ...	283	158	162	414	1017
	Totals	1054	462	458	1026	3000

Forming from this a 4-fold table on the principle stated in § 3, we have:—

3000 *Criminals.*

Height (feet and inches).

Head Breadth (cms.)		$5\cdot5\frac{9}{16}$ and under	over $5\cdot5\frac{9}{16}$	Totals
	15 and under	847	681	1528
	over *15* ...	669	803	1472
	Totals	1516	1484	3000

Here $\alpha_1 = \cdot010667$ $\alpha_2 = \cdot018667$.

Hence from the probability integral tables:

$$h = \cdot013366 \qquad k = \cdot023400$$

$$\log HK = \bar{1}\cdot2016623, \qquad \epsilon = \cdot156822,$$

and the equation in θ, where r, the coefficient of correlation, $= \sin\theta$, is:

$$\cdot156822 = \theta + \tfrac{1}{2}(\cdot000313)\,\theta^2 - (\cdot000727)\,\frac{\theta^3}{6} + (\cdot000313)\,\frac{5\theta^4}{24}.$$

$\theta = \cdot156822 = 8^\circ\,59'\,7''$ gives a very close solution, and therefore $r = \cdot1562$, the probable error being $\pm\,\cdot0190$.

On comparing this with the result obtained by the ordinary method, viz. $r = \cdot1831 \pm \cdot0119$, we conclude that we might improve our result by taking other limits of division; we therefore re-form our table as follows:

Height (feet and inches).

Head Breadth (cms.)		$5\cdot7\frac{9}{16}$ and under	over $5\cdot7\frac{9}{16}$	Totals
	15·3 and under	1799	397	2196
	over *15·3* ...	587	217	804
	Totals	2386	614	3000

Here
$$\alpha_1 = \cdot590667 \qquad \alpha_2 = \cdot464000$$
$$h = \cdot82507 \qquad k = \cdot61889$$
$$\log HK = \bar{2}\cdot9708265 \qquad \epsilon = \cdot186974$$

and the equation in θ is:

$$\cdot186974 = \theta + \cdot255314\theta^3 - \cdot133837\theta^3 + \cdot044029\theta^4.$$

Approximating to the root by Newton's rule, we find:

$$\theta = \cdot1794814 = 10^\circ\ 17'\ 1'' \text{ and } r = \cdot1785,$$

a better result than before, but we will try yet another division, viz.

TABLE X.

Height (feet and inches).

Head Breadth (cms.)		$5\cdot4\frac{9}{16}$ and under	over $5\cdot4\frac{9}{16}$	Totals
	14·8 and under	455	622	1077
	over *14·8* ...	599	1324	1923
	Totals	1054	1946	3000

Here
$$\alpha_1 = -\cdot297333 \qquad \alpha_2 = -\cdot282000$$
$$h = -\cdot38173 \qquad k = -\cdot36114$$
$$\log HK = \bar{1}\cdot1420742 \qquad \epsilon = \cdot184125.$$

The equation in θ now becomes:

$$\cdot184125 = \theta + \cdot068929\theta^3 - \cdot042689\theta^3 + \cdot024088\theta^4,$$

a solution of which is $\theta = \cdot1820710 = 10^\circ\ 25'\ 55''$,

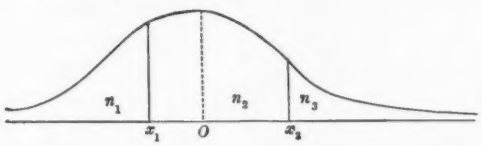
and $r = \cdot1811$

with a probable error of $\pm\ \cdot0210$.

As this result coincides very closely with that obtained by the usual method, we will rest satisfied with it, and in subsequent tables adopt the divisions of

Height and Head Breadth on which it is based. Comparing it with the worst value obtained above, ·1562, we see that even in that case the difference is only of the order ·02, or of the probable error of sampling.

(9) I have next to find the standard deviations and means of the two characters; this I shall do by applying the method explained in Professor Pearson and Dr Lee's memoir "On the inheritance of characters not capable of exact quantitative measurement" (*Phil. Trans.* Vol. 195, A., p. 82). Let us suppose our 16-fold table reduced to a 9-fold table, and let n_1, n_2, n_3, the three classes of one character which are thus formed, be represented by the areas of the normal curve in the accompanying diagram.



In all our investigations, the mean will fall within the area n_2. Let $Ox_1 = p_1$, $Ox_3 = p_3$, be the distances of the mean from the boundaries of n_2; then $p_1 + p_3$ in absolute measurement is known, $= \eta$, say, being the range of the class n_2.

Remembering that the equation to the normal curve is of the form

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}},$$

where N is the total frequency and σ the S.D., we have

$$\frac{n_2 + n_3 - n_1}{N} = \sqrt{\frac{2}{\pi}} \int_0^{h_1} e^{-\frac{1}{2}x^2}\, dx,$$

$$\frac{n_1 + n_2 - n_3}{N} = \sqrt{\frac{2}{\pi}} \int_0^{h_3} e^{-\frac{1}{2}x^2}\, dx,$$

where
$$h_1 = \frac{p_1}{\sigma}, \quad h_3 = \frac{p_3}{\sigma}.$$

Now the left-hand side is known, and from a table of the areas of the normal curve h_1 and h_3 are found. Therefore as $\sigma = \dfrac{p_1 + p_3}{h_1 + h_3} = \dfrac{\eta}{h_1 + h_3}$, the standard deviation is known. Also, as $p_1 = h_1\sigma$, the distance of the mean from the left-hand boundary of n_2, and therefore the mean itself, are known.

To find the probable error of σ, we have

$$\sigma = \frac{\eta}{h_1 + h_3}, \quad \therefore \delta\sigma = -\frac{\eta\,(\delta h_1 + \delta h_3)}{(h_1 + h_3)^2}.$$

$$\therefore \Sigma^2_\sigma = \frac{\eta^2}{(h_1 + h_3)^4} (\Sigma^2_{h_1} + \Sigma^2_{h_3} + 2\Sigma_{h_1}\Sigma_{h_3} R_{h_1 h_3}).$$

Now $$\frac{N-2n_1}{2N}=\frac{1}{\sqrt{2\pi}}\int_0^{h_1}e^{-\frac{1}{2}x^2}\,dx;$$

therefore, if $$H=\frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}h^2},$$

we have $$-\delta n_1=NH_1\delta h_1 \text{ and } \Sigma_{h_1}=\Sigma_{n_1}/(NH_1).$$

Similarly $$-\delta n_3=NH_3\delta h_3 \text{ and } \Sigma_{h_3}=\Sigma_{n_3}/(NH_3).$$

Also $\Sigma_{h_1}\Sigma_{h_3}R_{h_1h_3}=\Sigma_{n_1}\Sigma_{n_3}R_{n_1n_3}/N^2H_1H_3$, and it is shown in Professor Pearson's memoir "On the correlation of characters not quantitatively measurable" (*Phil. Trans.* Vol. 195, A., p. 11), that

$$\Sigma^2_{n_1}=\frac{n_1(N-n_1)}{N},\qquad \Sigma^2_{n_3}=\frac{n_3(N-n_3)}{N},$$

$$\Sigma_{n_1}\Sigma_{n_3}R_{n_1n_3}=-\frac{n_1n_3}{N},$$

therefore $$\Sigma^2_{h_1}=\frac{n_1(N-n_1)}{N^3H^2_1},\qquad \Sigma^2_{h_3}=\frac{n_3(N-n_3)}{N^3H^2_3},$$

$$2\Sigma_{h_1}\Sigma_{h_3}R_{h_1h_3}=-\frac{2n_1n_3}{N^3H_1H_3}.$$

The probable error of σ is accordingly $\cdot 67449\Sigma_\sigma$,

$$=\cdot 67449\,\frac{\eta}{(h_1+h_3)^2}\left\{\frac{n_1(N-n_1)}{N^3H^2_1}+\frac{n_3(N-n_3)}{N^3H^2_3}-\frac{2n_1n_3}{N^3H_1H_3}\right\}^{\frac{1}{2}}.$$

We have next to find the probable error in p_1, that is, the probable error of the mean.

$$p_1=h_1\sigma,$$

$$\delta p_1=h_1\delta\sigma+\sigma\delta h_1.$$

Therefore $$\Sigma^2_{p_1}=h^2_1\Sigma^2_\sigma+\sigma^2\Sigma^2_{h_1}+2h_1\sigma\Sigma_\sigma\Sigma_{h_1}R_{\sigma h_1};$$

also $$h_1+h_3=\frac{\eta}{\sigma},$$

$$\delta h_3=-\delta h_1-\eta\,\frac{\delta\sigma}{\sigma^2},$$

$$\Sigma^2_{h_3}=\Sigma^2_{h_1}+\frac{\eta^2}{\sigma^4}\Sigma^2_\sigma+\frac{2\eta}{\sigma^2}\Sigma_\sigma\Sigma_{h_1}R_{\sigma h_1}.$$

Substituting for $\Sigma_\sigma\Sigma_{h_1}R_{\sigma h_1}$ from the first equation, and remembering that $\eta=\sigma\,(h_1+h_3)$, we have finally, after a few reductions,

$$\Sigma^2_{p_1}=\frac{\sigma^2(h_1\Sigma^2_{h_3}+h_3\Sigma^2_{h_1})}{h_1+h_3}-\Sigma^2_\sigma h_1h_3,$$

and the probable error $=\cdot 67449\Sigma_{p_1}$.

(10) We will now apply these formulæ.

Height and Head Breadth. We first form the 9-fold Table XI. (p. 222).

Taking Height first, we have:

$$n_1 = 1054, \quad n_2 = 920, \quad n_3 = 1026.$$
$$h_1 = \cdot38173, \quad h_2 = \cdot40702, \quad \eta = 2.$$

Therefore
$$\sigma = \frac{2}{\cdot78875} = 2\cdot53566 \text{ inches},$$
$$p_1 = \sigma \times h_1 = \cdot967937.$$

The mean is therefore
$$64\tfrac{9}{16} + \cdot967937 = 65\cdot5304 \text{ inches}.$$

For Head Breadth
$$n_1 = 1077, \quad n_2 = 906, \quad n_3 = 1017,$$
$$h_1 = \cdot36114, \quad h_3 = \cdot415198, \quad \eta = \cdot4,$$
$$\sigma = \frac{\cdot4}{\cdot77634} = \cdot515238 \text{ (cm.)},$$
$$p_1 = \sigma \times h_1 = \cdot186073.$$

In finding the mean it is to be noted that the lower boundary of n_2 is 14·85 as the measurements are taken to the nearest millimetre; the mean is therefore
$$14\cdot85 + \cdot186073 = 15\cdot0361 \text{ cm}.$$

The probable errors of the S.D. are

Height	$\pm$ ·0495
Head Breadth	$\pm$ ·01015.

Height and Left Middle Finger. Our 4-fold table is Table XII., p. 222 (see also Table III., p. 216).
$$h = -\cdot38173, \quad k = \cdot01169, \quad \epsilon = \cdot714334.$$

The equation in θ is:
$$\cdot714334 = \theta - \cdot002232\theta^3 - \cdot024306\theta^3 - \cdot000849\theta^4 - \cdot000919\theta^5,$$
whence
$$\theta = \cdot7251964, \quad r = \cdot6633.$$

The 9-fold table is Table XIII., p. 222 (see also Table III., p. 216).

For Finger
$$n_1 = 706, \quad n_2 = 1612, \quad n_3 = 682,$$
$$h_1 = \cdot7214, \quad h_3 = \cdot74769, \quad \eta = \cdot8,$$
$$\sigma = \frac{\cdot8}{1\cdot46909} = \cdot544555,$$
$$p_1 = h_1\sigma = \cdot392842.$$

Therefore the mean is

$$11{\cdot}15 + {\cdot}392842 = 11{\cdot}542842,$$

and the probable error of $\sigma = {\cdot}0075$.

Head Length and Head Breadth. The 4-fold table is Table XIV., p. 222 (see also Table I., p. 214).

$$h = -{\cdot}03761\,;\ k = -{\cdot}36114\,;\ \epsilon = {\cdot}408616.$$

$$\cdot 408619 = \theta + {\cdot}006791\theta^2 - {\cdot}021942\theta^3 + {\cdot}002606\theta^4 - {\cdot}008325\theta^5,$$

whence $\theta = {\cdot}4090070, \quad r = {\cdot}3977.$

For S.D. of Head Length, we have from the 9-fold Table XV., p. 222,

$$n_1 = 463, \quad n_2 = 2043, \quad n_3 = 494,$$

$$h_1 = 1{\cdot}018019, \quad h_3 = {\cdot}975451, \quad \eta = 1{\cdot}2.$$

$$\sigma = \frac{1{\cdot}2}{1{\cdot}99347} = {\cdot}6019654,$$

and $p_1 = h_1\sigma = {\cdot}6128122,$

therefore $\text{Mean} = 18{\cdot}55 + {\cdot}6128122 = 19{\cdot}1628122,$

and the probable error of σ is ·0071.

(11) *Comparison of Results.* We can now compare the results obtained by the new method with those obtained from the full correlation tables:

TABLE 16.

Coefficient of Correlation.

	Old Method	New Method
Head Length and Head Breadth	·4016 ± ·0103	·3977 ± .0176
Head Breadth and Height ...	·1831 ± ·0119	·1811 ± ·0210
L. M. Finger and Height ...	·6608 ± ·0069	·6633 ± .0142

TABLE 17.

S.D. and Mean.

	Standard Deviation		Mean	
	Old Method	New Method	Old Method	New Method
Head Length (cm.)	·6046 ± ·0053	·6020 ± ·0071	19·1663 ± ·0075	19·1628 ± ·0086
Head Breadth (cm.)	·5014 ± ·0044	·5152 ± ·0101	15·0442 ± ·0062	15·0361 ± ·0072
Finger (cm.) ...	·5479 ± ·0048	·5446 ± ·0075	11·5474 ± ·0068	11·5428 ± ·0075
Height (ins.) ...	2·5410 ± ·0221	2·5357 ± ·0495	65·5355 ± ·0313	65·5304 ± ·0352

The extremely close agreement of the results thus obtained in two very different ways, especially the agreement in the means, is deserving of special attention and is a striking illustration of the value of the new method.

(12) *Application of Methods, continued.* I will now proceed to calculate the remaining standard deviations, means, and coefficients of correlation.

Head Length and Face Breadth. See Tables XVI. and XVII., p. 223.

$$h = -\cdot03761\,; \quad k = 0\,; \quad \epsilon = \cdot405552,$$

$$\cdot405552 = \theta - \cdot001415\frac{\theta^3}{6}\,;$$

whence $\theta = \cdot4055676$ and $r = \cdot3945 \pm \cdot0172$.

For S.D. and Mean of Face Breadth:

$$n_1 = 1039, \quad n_2 = 922, \quad n_3 = 1039, \quad h_1 = h_3 = \cdot39524\,;$$

therefore $$\sigma = \cdot5060216 \pm \cdot009856.$$

$$p_1 = h_1\sigma = \cdot2000, \text{ therefore Mean} = 13\cdot45 + \cdot2 = 13\cdot65.$$

Head Length and Cubit. Tables XVIII. and XIX., p. 223.

$$h = -\cdot03761, \quad k = -\cdot067734, \quad \epsilon = \cdot310452,$$

$$\cdot310452 = \theta + \cdot001274\theta^2 - \cdot001\theta^3 + \cdot000529\theta^4\,;$$

whence $$\theta = \cdot3103547\,; \quad r = \cdot3054.$$

For S.D. and Mean of Cubit:

$$n_1 = 911, \quad n_2 = 1002, \quad n_3 = 1087, \quad h_1 = \cdot51389, \quad h_3 = \cdot35224,$$

whence $$\sigma = 1\cdot96275 \pm \cdot0171.$$

Also $p_1 = h_1\sigma = 1\cdot00864$; therefore Mean $= 44\cdot05 + p_1 = 45\cdot05864$.

Head Length and Foot. Tables XX. and XXI., pp. 223, 224.

$$h = -\cdot03761, \quad k = -\cdot10295, \quad \epsilon = \cdot345875,$$

$$\cdot345875 = \theta + \cdot001936\theta^2 - \cdot002\theta^3 + \cdot000801\theta^4\,;$$

whence $$\theta = \cdot3457144, \quad r = \cdot3389.$$

For S.D. and Mean of Foot:

$$n_1 = 883, \quad n_2 = 979, \quad n_3 = 1138, \quad h_1 = \cdot540778, \quad h_3 = \cdot307234,$$

whence $$\sigma = 1\cdot1792286 \pm \cdot0103.$$

Also $p_1 = h_1\sigma = \cdot6377009$; therefore Mean $= 25\cdot05 + p_1 = 25\cdot6877$.

The divisions chosen for Foot are not quite satisfactory, the class "over 26" being too large.

Having now found the S.D. and Mean of all seven characters, we no longer require 9-fold tables, and we shall therefore now use only 4-fold tables in order to find r.

Head Length and Finger. Table XXII., p. 224.

$$h_1 = -\cdot 03761, \quad k = \cdot 01169, \quad \epsilon = \cdot 305411,$$
$$\cdot 305411 = \theta - \cdot 00022\theta^2 - \cdot 000259\theta^3 - \cdot 000091\theta^4;$$
whence $$\theta = \cdot 3054401, \qquad r = \cdot 3007 \pm \cdot 0181.$$

Head Length and Height. Table XXIII., p. 224.

$$h = -\cdot 38173, \quad k = -\cdot 03761, \quad \epsilon = \cdot 34673,$$
$$\cdot 34673 = \theta + \cdot 007178\theta^2 - \cdot 024479\theta^3 + \cdot 002727\theta^4;$$
whence $$\theta = \cdot 3468384, \qquad r = \cdot 3399 \pm \cdot 0207.$$

Head Breadth and Face Breadth. Table XXIV., p. 224.

$$h = -\cdot 36114, \quad k = 0, \quad \epsilon = \cdot 660597,$$
$$\cdot 660597 = \theta - \cdot 021737\theta^3 + \cdot 008269\theta^5;$$
whence $$\theta = \cdot 6659329, \qquad r = \cdot 6178.$$

If we calculate r from a division nearly median (Table XXIV.^{a}, p. 224) we have:

$$h = \cdot 0234, \quad k = 0, \quad \epsilon = \cdot 655725,$$
$$\cdot 655725 = \theta - \cdot 0000913\theta^3 - \cdot 0000365\theta^5;$$
whence $$\theta = \cdot 655755, \qquad r = \cdot 6098.$$

Head Breadth and Finger. Table XXV., p. 225.

$$h = -\cdot 36114, \quad k = \cdot 01169, \quad \epsilon = \cdot 15085,$$
$$\cdot 15085 = \theta - \cdot 002111\theta^2 - \cdot 021757\theta^3 - \cdot 000811\theta^4;$$
whence $$\theta = \cdot 1509732, \qquad r = \cdot 1504.$$

If we adopt a median division (Table XXV.^{a}, p. 225) we shall find $r = \cdot 1459$.

Head Breadth and Cubit. Table XXVI., p. 225.

$$h = -\cdot 36114, \quad k = -\cdot 06773, \quad \epsilon = \cdot 135737,$$
$$\cdot 135737 = \theta + \cdot 012230\theta^2 - \cdot 022402\theta^3 + \cdot 004684\theta^4;$$
whence $$\theta = \cdot 1355670, \quad r = \cdot 1352 \pm \cdot 0196.$$

Head Breadth and Foot. Table XXVII., p. 225.

$$h = -\cdot 10295, \quad k = -\cdot 36114, \quad \epsilon = \cdot 208238,$$
$$\cdot 208238 = \theta + \cdot 01859\theta^2 - \cdot 023273\theta^3 + \cdot 007092\theta^4;$$
whence $$\theta = \cdot 2076318, \qquad r = \cdot 2061.$$

Face Breadth and Finger. Table XXVIII., p. 225.

$$h = 0, \quad k = \cdot 01169, \quad \epsilon = \cdot 326748,$$
$$\cdot 326748 = \theta - \cdot 000023\theta^3,$$
whence $$\theta = \cdot 3267488, \qquad r = \cdot 3210.$$

Face Breadth and Cubit. Table XXIX., p. 226.

$$h = 0, \quad k = -\cdot 06773, \quad \epsilon = \cdot 292839,$$

$$\cdot 292839 = \theta - \cdot 0007647\theta^3,$$

whence $\theta = \cdot 2928580, \qquad r = \cdot 2887.$

Face Breadth and Foot. Table XXX., p. 226.

$$h = 0, \quad k = -\cdot 102952, \quad \epsilon = \cdot 371625,$$

$$\cdot 371625 = \theta - \cdot 0017665\theta^3,$$

whence $\theta = \cdot 3717160, \qquad r = \cdot 3632.$

Face Breadth and Height. Table XXXI., p. 226.

$$h = 0, \quad k = -\cdot 38173, \quad \epsilon = \cdot 351419,$$

$$\cdot 351419 = \theta - \cdot 024286\theta^3 - \cdot 009185\theta^5;$$

whence $\theta = \cdot 3525333, \qquad r = \cdot 3453.$

Foot and Finger. Table XXXII., p. 226.

$$h = -\cdot 10295, \quad k = \cdot 01169, \quad \epsilon = \cdot 859268,$$

$$\cdot 859268 = \theta - \cdot 0006020\theta^2 - \cdot 001789\theta^3 - \cdot 000249\theta^4 - \cdot 000713\theta^5,$$

whence $\theta = \cdot 8613318, \qquad r = \cdot 7587.$

Finger and Cubit. Table XXXIII., p. 226.

$$h = \cdot 01169, \quad k = -\cdot 06773, \quad \epsilon = 1\cdot 007431,$$

$$1\cdot 007431 = \theta - \cdot 000396\theta^2 - \cdot 000787\theta^3 - \cdot 000164\theta^4 - \cdot 000314\theta^5;$$

whence $\theta = 1\cdot 0091435, \qquad r = \cdot 8464 \pm \cdot 0079.$

As the cubit includes the finger, their correlation may be considered "spurious." Let us define "forearm" as the cubit minus the finger; then we can find S.D. and mean of forearm, and correlation of forearm and finger, as follows:—

Let a = forearm, c = cubit, f = finger.

$$c = a + f \text{ and } cf = af + f^2.$$

Therefore $$S(cf) = S(af) + S(f^2),$$

or $$r_{fc}\sigma_f\sigma_c = r_{af}\sigma_a\sigma_f + \sigma^2_f,$$

therefore $$r_{af} = \frac{r_{fc}\sigma_c - \sigma_f}{\sigma_a},$$

which gives the correlation of forearm and finger. Substituting the known values of the symbols on the right-hand side we have:

$$r_{af} = \cdot 7297.$$

Also $a = c - f$, therefore $\sigma_a^2 = \sigma_c^2 + \sigma_f^2 - 2\sigma_c\sigma_f r_{cf}$,

and substituting we find $\sigma_a = 1{\cdot}5297$,

and $\text{Mean}_a = \text{Mean}_c - \text{Mean}_f = 33{\cdot}5158$.

Left Foot and Left Cubit. Table XXXIV., p. 227.

$$h = -{\cdot}10295, \quad k = -{\cdot}06773, \quad \epsilon = {\cdot}923660,$$

$$\cdot 923660 = \theta + {\cdot}003486\theta^2 - {\cdot}002523\theta^3 + {\cdot}001439\theta^4 - {\cdot}0009986\theta^5;$$

whence $\theta = {\cdot}9222992; \quad r = {\cdot}79699.$

Height and Left Cubit. Table XXXV., p. 227.

$$h = {\cdot}01337, \quad k = -{\cdot}06773, \quad \epsilon = {\cdot}925690,$$

$$\cdot 925690 = \theta - {\cdot}000453\theta^2 - {\cdot}000794\theta^3 - {\cdot}000188\theta^4 - {\cdot}000317\theta^5;$$

whence $\theta = {\cdot}9270676; \quad r = {\cdot}7999.$

Here we have adopted the median division. If we work with the skew division, Table XXXV.[a], p. 227, we have:

$$h = -{\cdot}38173, \quad k = -{\cdot}06773, \quad \epsilon = {\cdot}910952,$$

$$\cdot 910952 = \theta + {\cdot}012927\theta^2 - {\cdot}024939\theta^3 + {\cdot}004901\theta^4 - {\cdot}009349\theta^5;$$

whence $\theta = {\cdot}9222068; \quad r = {\cdot}7969;$

but it will be observed that in the second equation for θ the terms converge much more slowly than in the first; we will therefore retain the first value, especially as the difference between the two is only ·00292.

Height and Left Foot. Table XXXVI., p. 227.

$$h = -{\cdot}013366, \quad k = -{\cdot}102952, \quad \epsilon = {\cdot}82577,$$

$$\cdot 825770 = \theta - {\cdot}000688\theta^2 - {\cdot}001796\theta^3 - {\cdot}000285\theta^4 - {\cdot}0007153\theta^5;$$

whence $\theta = {\cdot}8276709; \quad r = {\cdot}7364.$

Here again we have adopted the median division.

Adopting the skew division, Table XXXVI.[a], p. 227, we have:

$$h = -{\cdot}38173, \quad k = -{\cdot}10295, \quad \epsilon = {\cdot}881436,$$

$$\cdot 881436 = \theta + {\cdot}01965\theta^2 - {\cdot}025795\theta^3 + {\cdot}007422\theta^4 - {\cdot}009565\theta^5;$$

whence $\theta = {\cdot}8845487; \quad r = {\cdot}7736.$

It will be noticed that when r is large, the terms involving the higher powers of θ do not always converge as rapidly as we should like in the equations for θ; the values of θ are therefore not to be relied on beyond the 4th figure or so. To go up to the 6th power of θ would considerably increase the labour of solving the equations; we have therefore not gone beyond the 5th power, the term involving which is:

$$+[h^4k^4 + (h^2 + k^2)\{3(h^2 + k^2) - 6h^2k^2 - 8\} + 20h^2k^2]\frac{\theta^5}{120}.$$

(13) For convenience of reference, we will now collect all the standard deviations, means, and coefficients of correlation as ascertained in the course of the preceding inquiry; also the coefficients of variation.

TABLE 18.

3000 *Criminals.*

	Standard Deviation	Coefficient of Variation	Mean	
Head Length (cms.) ...	·6046	3·1544	19·1663	Table 1
Head Breadth (cms.) ...	·5014	3·3332	15·0442	Table 1
Face Breadth (cms.) ...	·5060	3·7071	13·6500	§ 12
Left Middle Finger (cms.)	·5479	4·7445	11·5474	Table 1
Left Cubit (cms.) ...	1·9627	4·3560	45·0586	§ 12
Left Foot (cms.) ...	1·1792	4·5906	25·6877	§ 12
Height (ins.)	2·5410	3·8773	65·5355	Table 1
„ (cms.)	6·4540		166·4572	

TABLE 19.

Coefficients of Correlation. 3000 *Criminals.*

	Head Length	Head Breadth	Face Breadth	Finger	Cubit	Foot	Height
Head Length	1·	·40163	·39454	·30071	·30539	·33886	·33993
Head Breadth	·40163	1·	·61779	·15040	·13515	·20614	·18308
Face Breadth	·39454	·61779	1·	·32097	·28869	·36322	·34527
Finger ...	·30071	·15040	·32097	1·	·84638	·75871	·66084
Cubit ...	·30539	·13515	·28869	·84638	1·	·79699	·79986
Foot	·33886	·20614	·36322	·75871	·79699	1·	·73636
Height ...	·33993	·18308	·34527	·66084	·79986	·73636	1·

(14) It now becomes necessary to test the probable errors of some of these results.

TABLE 20.

Probable Errors of the Means in Criminals.

	By Full Correlation Table of 3000	By Full Correlation Table of 1306	By New Method	$h_1 + h_3$
Head Length ...	·0075	·0112	·0086	1·99347
Head Breadth ...	·0062	·0096	·0072	·77634
Face Breadth ...	—	·0094	·0070	·79048
Finger	·0067	·0102	·0075	1·46909
Height	·0313	·0484	·0352	·78875

TABLE 21.

Probable Errors of S.D. in Criminals.

	By Full Correlation Table, 3000	By Full Correlation Table, 1306	By New Method, 3000	h_1+h_3
Head Length (cms.)	·0053	·0079	·0071	1·99347
Head Breadth (cms.)	·0044	·0068	·0101	·77634
Face Breadth (cms.)	—	·0066	·0099	·79048
Finger (cms.) ...	·0048	·0072	·0075	1·46909
Height (ins.) ...	·0221	·0342	·0495	·78875

TABLE 22.

Probable Errors of r in Criminals.

	By New Method	By Formula $\frac{\cdot 6745\,(1-r^2)}{\sqrt{1100}}$
Height and Head Breadth ($r=\cdot 1831$)	·0210	·0197
Head Length and Face Breadth ($r=\cdot 3945$)	·0172	·0172
Head Length and Finger ($r=\cdot 3007$)	·0181	·0185
Head Length and Height ($r=\cdot 3399$)	·0207	·0180
Head Breadth and Cubit ($r=\cdot 1352$)	·0196	·0200
Finger and Cubit ($r=\cdot 8464$)	·0079	·0058

It will be noted from the results for Head Length and Finger that the Probable Errors of the standard deviations by the new method for 3000 are very nearly equal to those by the usual method for 1306 when h_1+h_3 is large (see § 9), that is, when we arrange our 9-fold table so that the middle division is large. A reference to § 11 will also show that by arranging our tables in this way we obtained remarkably good results for S.D. and mean of Head Length and Finger; we might therefore expect improved results if we were to arrange all our 9-fold tables throughout on this principle.

It is to be noted that the Probable Errors of the means by the new method are practically as small as those of the standard deviations.

Owing to the laborious character of the calculation for finding the probable error of r by the new method, I have worked out only eight of the 21 errors (see Tables 16 and 22), but these eight being fairly representative will give a good idea of the magnitudes involved.

In the 2nd column of Table 22, the probable error is calculated by the usual formula for normal frequency, supposing $n=1100$, and on the whole the results correspond very closely with those obtained by the new method, except in the case of the last coefficient, where the difference is considerable, but as it happens the absolute amount of error is small in either case. We may therefore

conclude that by applying the new method to 3000 cases we shall obtain the coefficients of correlation to the same degree of accuracy as if we calculated them from a full correlation table of 1100 cases.

As a result of the preceding investigation I conclude that the new method (i) saves a great deal of time and labour; (ii) gives excellent results for standard deviation and mean, especially if the middle division of the 9-fold table is made comparatively large; and (iii) gives less good results for r when r is high, owing to the slowness of convergence of the terms in the equation for θ, but on the whole it enables us to calculate r with an accuracy fairly comparable to that attained from an ordinary frequency table containing about one-third the number of frequencies.

(15) I will now state some results, based on Tables 18 and 19, which I think are sufficiently novel and important to be emphasised. I note that the determinations of Left Foot, so far as I can ascertain, are the first that have been made, and have therefore the interest of novelty.

The following table shows how the mean head measurements of criminals compare with those of the British Association and the University College series which are given by Dr Alice Lee on page 251 of her memoir, "A first study of the Correlation of the Human Skull" (*Phil. Trans.*, Vol. 196); the B.A. measurements are averages obtained by Dr Lee from the values given for several years in the "Reports of the B.A. Anthropometric Committee"; the University College measurements were taken by Professor Karl Pearson on twenty-five members of the College staff.

TABLE 23.

Mean Head Measurements.

	B. A.	U. C. Staff	Criminals
Length	198·1	196·38	191·7
Breadth	155·0	153·48	150·4

As regards variability, Dr Lee's memoir, p. 230, enables me to make the following comparison:

TABLE 24.

Coefficients of Variation.

	Head Length	Head Breadth
Aino Males ...	3·195	2·759
German Males ...	3·371	3·887
Criminals	3·154	3·333

And if we compare our coefficients of variation as a whole with those obtained for other parts of the body by various investigators, we shall find a general agreement in the results.

For instance, Miss Whiteley has shown that the coefficients of variation for the first joints of the fingers range from 4·695 to 5·361 (*Proceedings of the Royal Society*, Vol. 65, p. 129); and Professor Pearson has found that these coefficients for the long bones (Aino and French) range from 4·655 to 5·425.

From these figures it appears that 3 to 5·5 are representative values for variability in man, while in plants it may run to 40!

I may direct attention to some of the general results of the table of correlation (Table 19); e.g., the close relationship of finger and foot, of cubit and foot, which I think is novel and interesting, of height and finger, and cubit and finger. It will be noticed that the high correlations between finger, cubit, foot and height agree generally with the correlations between height and long bones as calculated by Professor Pearson (*Phil. Trans.* Vol. 192, A., p. 181, Table III.). For instance, he gives the coefficient of correlation between height and radius as ·6956, as compared with ·7999 the coefficient of correlation between height and cubit in criminals.

Turning now to the head measurements, Head Length, Head Breadth and Face Breadth, we observe that our results confirm the view that parts of the head are less highly correlated together than other parts of the body. It is also to be noticed that comparatively little correlation exists between the head and the other four characters, and that Head Length is more closely correlated with Cubit than is Head Breadth or Face Breadth, while Face Breadth is more nearly correlated with Foot than is Head Length or Head Breadth. Long head associated with long arm (i.e. cubit) rather suggests the gorilla type, but I can think of no type suggested by the association of long foot and broad face.

PART III.

APPLICATION OF THESE RESULTS TO CRIMINAL IDENTIFICATION.

(16) The conditions to be satisfied in selecting a series of organs for criminal identification are:

(i) Comparative ease and accuracy of measurement.

(ii) Small correlation between them, so that fairly few organs will provide a reliable index to the criminal population.

Head Length, Head Breadth and Face Breadth can be fairly accurately determined; Cubit and Finger with less accuracy; Height changes about $\frac{1}{8}''$ per 10 years from about 27 years of age*, and is not very reliable; Foot is a doubtful measurement.

* *Biometrika*, Vol. I. p. 49.

Now the organs selected by Scotland Yard are very far from being slightly correlated. We have seen that Face Breadth and Head Breadth are highly correlated, and the correlations between Finger, Cubit, Foot and Height are very high. It cannot therefore be said that these seven measurements form an ideal group, and I do not suppose they would have been chosen if their correlations had been actually known beforehand. My results at this point conflict with those of Dr Garson, who writes in his paper in the *Journal of the Anthropological Institute* to which I have already referred: "In a mixed population such as we have to deal with in England, the correlation between the different measurements used for the classification of criminals is slight."

Supposing, however, these organs have been selected, we may ask in the next place: What is the best order for entering an Index? This is the problem to which we shall next turn, and for its solution I propose to apply a general theorem in correlation which may be stated thus:

(17) Given n variables, then the most probable value A_1 of one of the variables, for given values $A_2, A_3, \dots A_n$, of the remaining $n-1$, is given by the equation

$$A_1 - m_1 = -\sigma_1 \left\{ \frac{R_{12}}{R_{11}} \frac{(A_2 - m_2)}{\sigma_2} + \frac{R_{13}}{R_{11}} \frac{(A_3 - m_3)}{\sigma_3} + \dots + \frac{R_{1n}}{R_{11}} \frac{(A_n - m_n)}{\sigma_n} \right\},$$

and its variability $\bar{s}_1 = \sigma_1 \sqrt{\dfrac{\Delta}{R_{11}}}$, where σ_1 is the S.D. of the organ A_1, and σ_q of the organ A_q; m_1 the mean of A_1 and m_q the mean of A_q; r_{1q} the correlation coefficient of A_1 and A_q, and $r_{qq'}$ the correlation coefficient of A_q and $A_{q'}$; and finally Δ is the determinant

$$\begin{vmatrix} 1 & r_{12} & r_{13} & r_{14} & \dots & r_{1n} \\ r_{21} & 1 & r_{23} & . & & . \\ \vdots & . & . & . & & . \\ r_{n1} & r_{n2} & . & . & & 1 \end{vmatrix}$$

and R_{pq} the minor corresponding to r_{pq}.

When $A_2, A_3, \dots A_n$ are known for only one individual, the probable error of the determination A_1 is

$$\cdot 67449\, \sigma_1 \sqrt{\frac{\Delta}{R_{11}}};$$

when they are known for p individuals, the probable error of A_1, which is then the corresponding mean value, becomes

$$\cdot 67449\, \sigma_1 \sqrt{\frac{\Delta}{pR_{11}}}.$$

When $n = 2$, the reconstruction formula becomes

$$A_1 - m_1 = \frac{\sigma_1}{\sigma_2} r_{12} (A_2 - m_2),$$

and the probable error

$$\cdot 67449\,\sigma_1\sqrt{\frac{1-r^2_{12}}{p}}.$$

(See Pearson, *Phil. Trans.* Vol. 192, A., pp. 171–2.)

(18) In our case Δ is the determinant of the 7th order*:

$$\begin{vmatrix}
1\cdot & \cdot 40163 & \cdot 39454 & \cdot 30071 & \cdot 30539 & \cdot 33886 & \cdot 33993 \\
\cdot 40163 & 1\cdot & \cdot 61779 & \cdot 15040 & \cdot 13515 & \cdot 20614 & \cdot 18308 \\
\cdot 39454 & \cdot 61779 & 1\cdot & \cdot 32097 & \cdot 28869 & \cdot 36322 & \cdot 34527 \\
\cdot 30071 & \cdot 15040 & \cdot 32097 & 1\cdot & \cdot 84638 & \cdot 75871 & \cdot 66084 \\
\cdot 30539 & \cdot 13515 & \cdot 28869 & \cdot 84638 & 1\cdot & \cdot 79699 & \cdot 79986 \\
\cdot 33886 & \cdot 20614 & \cdot 36322 & \cdot 75871 & \cdot 79699 & 1\cdot & \cdot 73636 \\
\cdot 33993 & \cdot 18308 & \cdot 34527 & \cdot 66084 & \cdot 79986 & \cdot 73636 & 1\cdot
\end{vmatrix}$$

the value of which is found by the laborious process of reduction to be $= \cdot 012129$.

Also

$R_{11} = \cdot 016272$	and	$\sqrt{\Delta/R_{11}} = \cdot 86336$
$R_{22} = \cdot 020907$		$\sqrt{\Delta/R_{22}} = \cdot 76167$
$R_{33} = \cdot 022556$		$\sqrt{\Delta/R_{33}} = \cdot 73330$
$R_{44} = \cdot 047496$		$\sqrt{\Delta/R_{44}} = \cdot 50534$
$R_{55} = \cdot 071864$		$\sqrt{\Delta/R_{55}} = \cdot 41083$
$R_{66} = \cdot 040198$		$\sqrt{\Delta/R_{66}} = \cdot 54930$
$R_{77} = \cdot 038347$		$\sqrt{\Delta/R_{77}} = \cdot 56240.$

Now the best organ to leave to the last will be that the variability of which is least reduced by selecting the other six. For this means that the individuals from whom we have in the last instance to select our man, will be least crowded together and therefore least likely to be indistinguishable. As $\bar{s} = \sigma\sqrt{\frac{\Delta}{R}}$, the greater $\sqrt{\frac{\Delta}{R}}$, the less the reduction in variability; this function we have already calculated for $n = 7$, and we see that in that case it is greatest for Head Length. We will therefore keep Head Length to the last, and proceed to calculate $\sqrt{\frac{\Delta}{R}}$ for the six organs that remain after the elimination of Head Length. The new Δ is obviously $R_{11} = \cdot 016272$, and the new R's can be calculated without much trouble. On comparing the new $\sqrt{\frac{\Delta}{R}}$ series, we shall find that Head Breadth is the next character to be eliminated, and we proceed in like manner until we have dealt with all the characters.

* Values of the correlation coefficients are given to 5 places of figures in order to calculate the determinants more accurately.

The result of these operations is shown in the following table:

TABLE 25.

Showing the value of $\sqrt{\Delta/R}$ *for* 3, 4, 5, 6 *and* 7 *characters.*

	7	6	5	4	3
Head Length ...	·86336	—			
Head Breadth ...	·76167	·78408	—		
Face Breadth ...	·73330	·73763	·91541	—	
Finger	·50534	·50568	·50607	·50959	·51403
Cubit	·41083	·41083	·41089	·41452	·47660
Foot	·54930	·55016	·55060	·55671	·58296
Height	·56240	·56502	·56504	·57244	—

The order in which the various characters should be taken is therefore: 1st, Cubit; 2nd, Finger; 3rd, Foot; 4th, Height; 5th, Face Breadth; 6th, Head Breadth; 7th, Head Length, and is the same as that which is given by the first column, for $n = 7$. It is to be noted that the head measurements come last.

This result conflicts with the practice at Scotland Yard, where, I understand, the cabinet is entered in the order Head Length, Head Breadth, Face Breadth, Finger, Cubit, Foot and Height, the head measurements being taken first.

Should it be maintained that in fixing the order of the characters we should use $\bar{s}$, and not $\sqrt{\frac{\Delta}{R}}$, as the criterion, that is, that we should leave to the last the organ whose absolute variability is greatest, not the organ whose variability is least reduced, after selection of the other six, I have calculated the following table to show how this principle will work out:

TABLE 26.

Showing the value of $\bar{s} = \sigma\sqrt{\Delta/R}$ *for* 3, 4, 5, 6 *and* 7 *characters.*

	7	6	5	4	3
Head Length $\sigma =$ ·6046	·5220	·5244	·5251	·5289	—
Head Breadth $\sigma =$ ·5015	·3819	·3821	·3823	·3823	·3935
Face Breadth $\sigma =$ ·5060	·3711	·3738	·3741	·3769	·3804
Finger $\sigma =$ ·5479	·2769	·2801	·3543	·5051	·5178
Cubit $\sigma = 1{\cdot}9628$	·8064	·9323	—	—	—
Foot $\sigma = 1{\cdot}1792$	·6478	·6707	·7478	—	—
Height $\sigma = 6{\cdot}4540$	3·6297	—	—	—	—

In the case of height the unit of record is $\frac{1}{8}$th inch = 3·175 mm., for the other characters the unit is 1 mm.; we therefore divide the 3·62971 of the table by

3·175 to obtain a true comparison for height, and find $\bar{s} = 1{\cdot}1432$, which is still the greatest value of the 7 column.

On this principle, then, the order is 1st, Face Breadth; 2nd, Head Breadth; 3rd, Finger; 4th, Head Length; 5th, Foot; 6th, Cubit; and 7th, Height; it will be observed that this is the order of the standard deviations of the characters, but differs widely from the Scotland Yard order.

I think, however, that $\sqrt{\Delta/R}$ is the correct criterion. We cannot read height to the same accuracy as head length. $\bar{s}$ for height = 3·6297, and for head length ·5220, but head length is read to $\frac{1}{2}$ mm. and height to $\frac{1}{16}$ in. Now head length is hardly likely to be correct to more than 1 mm. or height to $\frac{1}{4}$ inch = 6 mm. say, or even to $\frac{1}{2}$ inch = 12 mm.; hence I doubt if stature with an $\bar{s}$ of 3·6 is really better than head length with a range of ·5. The right principle seems to be to suppose each organ on its own S.D. equally useful, and then judge its efficiency by the extent to which that S.D. is altered by selection of the other characters.

(19) Professor Pearson has pointed out to me that the ideal index characters would be given if we calculated the seven directions of uncorrelated variables, that is, the principal axes of the correlation "ellipsoid." Thus, given x_1, x_2, ... x_7 correlated variables, the seven uncorrelated variables are:

$$X_1 = l_{11}x_1 + l_{12}x_2 + \ldots + l_{17}x_7$$
$$X_2 = l_{21}x_1 + l_{22}x_2 + \ldots + l_{27}x_7$$
$$\text{\&c.} \qquad \text{\&c.}$$

where the l's give the directions of the principal axes, and X_1, X_2, ... X_7 would be the proper index functions to identify criminals by if we have nothing better than the present correlated characters to work with. Of course this necessitates a preliminary determination of 49 numerical multipliers, but if these were once calculated, the uncorrelated characters of a criminal could be easily found from the measured correlated characters, and his identity established from an X-cabinet, into which we might enter *in any order*. I propose to return in a later paper to this calculation.

(20) *Reconstruction of Height from a knowledge of Finger, Cubit, and Foot, for medico-legal purposes.*

I now propose to consider in greater detail the four characters which have very high correlation, viz., Finger, Cubit, Foot and Height. For these four,

$$\Delta = \begin{vmatrix} 1{\cdot} & {\cdot}84638 & {\cdot}75871 & {\cdot}66084 \\ {\cdot}84638 & 1{\cdot} & {\cdot}79699 & {\cdot}79986 \\ {\cdot}75871 & {\cdot}79699 & 1{\cdot} & {\cdot}73636 \\ {\cdot}66084 & {\cdot}79986 & {\cdot}73636 & 1{\cdot} \end{vmatrix} = {\cdot}031586,$$

and
$$R_{11} = \cdot121635, \quad \sqrt{\Delta/R_{11}} = \cdot509587, \quad R_{12} = \cdot088885$$
$$R_{22} = \cdot183825, \quad \sqrt{\Delta/R_{22}} = \cdot414520, \quad R_{13} = -\cdot031910$$
$$R_{33} = \cdot101914, \quad \sqrt{\Delta/R_{33}} = \cdot556710, \quad R_{14} = -\cdot014211$$
$$R_{44} = \cdot096392, \quad \sqrt{\Delta/R_{44}} = \cdot572436, \quad R_{23} = \cdot030695$$
$$R_{24} = -\cdot065693$$
$$R_{34} = \cdot029405$$

Using the values of S.D. and Means given in Table 18 we can now reconstruct the various characters if one or more are given us; we shall choose Height (H) as being probably the most useful example.

(i) *Reconstruction of Height from Finger.*

$$H = 166{\cdot}45716 + \frac{6{\cdot}45397}{{\cdot}54786} \times {\cdot}66084\ (\text{Finger} - 11{\cdot}54737),$$
$$= 166{\cdot}45716 + 7{\cdot}7849\ (\text{Finger} - 11{\cdot}54737).$$

Probable error $= {\cdot}67449 \times 6{\cdot}45397\ \sqrt{1 - ({\cdot}66084)^2}/\sqrt{p},$
$$= 3{\cdot}2671/\sqrt{p}.$$

(ii) *Reconstruction of Height from Cubit.*

$$H = 166{\cdot}45716 + 2{\cdot}6301\ (\text{Cubit} - 45{\cdot}05864).$$

Probable error $= 2{\cdot}6127/\sqrt{p}.$

(iii) *Reconstruction of Height from Foot.*

$$H = 166{\cdot}45716 + 4{\cdot}0301\ (\text{Foot} - 25{\cdot}68770).$$

Probable error $= 2{\cdot}9453/\sqrt{p}.$

(iv) *Reconstruction of Height from Finger and Cubit.*

In this case,

$$\Delta = \begin{vmatrix} 1{\cdot} & {\cdot}66084 & {\cdot}79986 \\ {\cdot}66084 & 1{\cdot} & {\cdot}84638 \\ {\cdot}79986 & {\cdot}84638 & 1{\cdot} \end{vmatrix} = {\cdot}101914,$$

$$R_{11} = {\cdot}28364, \quad R_{12} = {\cdot}01614, \quad R_{13} = -{\cdot}24054,$$
$$\sigma_2 = {\cdot}54786, \qquad\qquad \sigma_3 = 1{\cdot}96275,$$
$$m_2 = 11{\cdot}54737, \qquad\qquad m_3 = 45{\cdot}05864.$$

Therefore equation for H is:

$$H = 166{\cdot}45716 - {\cdot}6703\ (\text{Finger} - 11{\cdot}54737) + 2{\cdot}7886\ (\text{Cubit} - 45{\cdot}05864)^*$$

and probable error $= \dfrac{{\cdot}67449 \times 6{\cdot}45397}{\sqrt{p}} \sqrt{\dfrac{{\cdot}101914}{{\cdot}28364}},$
$$= 2{\cdot}6094/\sqrt{p}.$$

* This equation is of special interest, it shows that for persons of a given cubit the longer the finger, the less the height.

(v) *Reconstruction of Height from Finger and Foot.*

In this case,

$$\Delta = \begin{vmatrix} 1\cdot & \cdot66084 & \cdot73636 \\ \cdot66084 & 1\cdot & \cdot75871 \\ \cdot73636 & \cdot75871 & 1\cdot \end{vmatrix} = \cdot183825,$$

$$R_{11} = \cdot42436, \quad R_{12} = -\cdot10216, \quad R_{13} = -\cdot23497,$$

$$\sigma_2 = \cdot54786, \qquad \sigma_3 = 1\cdot17923,$$

$$m_2 = 11\cdot54737, \qquad m_3 = 25\cdot68770.$$

Therefore the equation becomes:

$$H = 166\cdot45716 + 2\cdot8360\ (\text{Finger} - 11\cdot54737) + 3\cdot0304\ (\text{Foot} - 25\cdot68770),$$

and probable error $= 2\cdot8651/\sqrt{p}$.

(vi) *Reconstruction of Height from Cubit and Foot.*

In this case,

$$\Delta = \begin{vmatrix} 1\cdot & \cdot79986 & \cdot73636 \\ \cdot79986 & 1\cdot & \cdot79699 \\ \cdot73636 & \cdot79699 & 1\cdot \end{vmatrix} = \cdot121635,$$

$$R_{11} = \cdot36481, \quad R_{12} = -\cdot21299, \quad R_{13} = -\cdot09888,$$

$$\sigma_2 = 1\cdot96275, \qquad \sigma_3 = 1\cdot17923,$$

$$m_2 = 45\cdot05864, \qquad m_3 = 25\cdot68770.$$

Therefore the equation is:

$$H = 166\cdot45716 + 1\cdot9198\ (\text{Cubit} - 45\cdot05864) + 1\cdot4834\ (\text{Foot} - 25\cdot6877),$$

and probable error $= 2\cdot5136/\sqrt{p}$.

(vii) *Reconstruction of Height from Finger, Cubit and Foot.*

The Δ and R's are already calculated, and from these we can form the equation in H. We will however calculate only the probable error, which is

$$\frac{\cdot67449 \times 6\cdot45397}{\sqrt{p}} \sqrt{\frac{\cdot031586}{\cdot096392}}$$

$$= 2\cdot4919/\sqrt{p}.$$

(viii) *Reconstruction of Height from all the other six organs* (see formula in § 17).

This formula has not been worked out numerically, but its probable error

$$= \frac{\cdot67449 \times 6\cdot45397}{\sqrt{p}} \sqrt{\frac{\cdot012129}{\cdot038347}}$$

$$= 2\cdot4482/\sqrt{p}.$$

We will now collect the probable errors which we have just calculated, taking $p = 1$, which is the case that most interests us.

TABLE 27.

Reconstruction of Height.

From	Probable Error
Finger	3·2671
Cubit	2·6127
Foot	2·9453
Finger and Cubit	2·6094
Finger and Foot	2·8651
Cubit and Foot	2·5136
Finger, Cubit and Foot	2·4919
Finger, Cubit, Foot, Head Length, Head Breadth and Face Breadth	2·4482

It will be noted that we do not improve much on the result obtained from Cubit and Foot by taking into account Finger also; nor on the result obtained from Cubit alone by taking into account Finger and Foot or even by the addition of all the remaining six characters.

As a practical problem, it is conceivable that a medico-legal case might arise where it would be important to discover the most probable height, given a foot or a cubit; the equations in (iii) and (ii) would enable us to reconstruct the height with a probable error of 2·9453 mm. and 2·6127 mm. respectively.

(21) *Examples of Reconstruction Formulæ.* I will illustrate the sort of errors that will be made by the use of these formulæ by applying them to the reconstruction of stature in ten cases selected at random from my material.

The Table 28 shows the difference between the actual and the calculated heights, *minus* denoting that the calculated stature is in defect, *plus* that it is in excess.

It will be noticed that the result of calculating from Finger and Cubit is practically the same as from Cubit alone.

As was to be expected, the reconstruction is only approximate, but it is quite as good as the results obtained for the reconstruction of stature from the long bones*. Given the Cubit, the Finger and Cubit or the Foot and Cubit, we can estimate the stature with a probable error of about one inch. We see that the actual ten cases give deviations, plus and minus, clustered round the probable error of each method of estimating in quite a reasonable manner. Occasionally our estimate might be wrong by upwards of two inches, but such cases are rare. It is of course the old story: No formula will give the stature with very great closeness from the measurement of one or two other organs. If it did, organic relationship would be perfect correlation, variation would have ceased, and with it of necessity the Darwinian theory of evolution. Allowing accordingly for this limitation, I believe the results obtained to be of use for purposes of criminal investigation. For example a finger, cubit or foot having been discovered, and

* Pearson: On the Reconstruction of the Stature of Prehistoric Races, *Phil. Trans.* Vol. 192, A., p. 188.

a crime being suspected, the police might from my formulæ give a very fair estimate of the stature of the supposed victim for the purpose of identification. The form of such an estimate from the cubit, for my first individual, for example, would be 62″ ± 1″.

TABLE 28.

Showing the difference in cms. between Actual and Calculated Height in 10 Criminals selected at random.

Actual Stature, cms.	Stature Calculated from						Average
	Finger	Cubit	Foot	Finger and Cubit	Finger and Foot	Foot and Cubit	
160·3	+ ·3	−2·4	+ ·6	−2·4	− ·2	−2·1	−1·03
168·3	−1·4	−2·3	− 6·2	−2·3	−5·	−3·8	−3·50
170·2	−2·6	−4·2	−3·3	−4·3	−3·	−3·9	−3·55
158·4	+5·4	+1·3	−2·	+1·2	− ·5	− ·5	+ ·82
161·9	+1·1	+3·1	+2·6	+3·3	+1·8	+2·8	+2·45
165·7	+ ·3	− ·4	+3·2	− ·5	+2·5	+ ·8	+ ·98
167·6	+1·6	+ ·5	+5·8	+ ·4	+5·	+2·6	+2·65
160·	+4·5	+1·	+3·7	+ ·9	+3·7	+1·5	+2·55
161·9	+8·1	+6·2	− ·6	+6·	+1·9	+3·9	+4·25
169·5	−5·	−1·1	−4·2	− ·8	−4·6	−2·	−2·95
Average	3·03	2·25	3·22	2·21	2·82	2·39	2·47
Prob. Error	±3·3	±2·6	±2·9	±2·6	±2·9	±2·5	±2·8

(22) *Summary.* I will conclude with a short summary of the results set forth in detail in this paper.

(i) The great value of the new method of calculating standard deviations, means, and coefficients of correlation is demonstrated; it economises labour, and leads to results closely approximating to those obtained by more laborious methods.

(ii) Then as regards the criminals; they are shown to be homogeneous, fairly normal in the distribution of the selected characters, markedly different from the educated classes in stature, and in size and shape of the head, but in variability they agree generally with other classes and races of mankind.

(iii) Finally as regards the problem of identification, it has been shown that certain characters at present in use are not very suitable for the purpose, owing to their high correlation, but assuming them to be employed, we have discovered the proper order in which they should be entered in an index cabinet—an order very different from that determined by Scotland Yard—and have indicated a method of calculating uncorrelated characters which would furnish an ideal system of identification.

TABLE I. 3000 *Criminals.*

Head Length (centimetres).

Head Breadth (centimetres).	16·						17·									18·										19·										20·										21·				Totals
	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	
13·4	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
5	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
6	–	–	–	–	–	–	–	–	–	1	1	–	–	–	1	1	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	5
7	–	–	–	–	–	–	–	–	1	1	–	–	–	–	1	–	1	–	–	–	–	–	–	1	–	–	1	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	7
8	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	3	–	–	–	–	1	–	–	–	–	1	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	8
9	–	–	–	–	–	–	1	–	–	–	–	1	1	1	1	1	–	–	–	–	1	–	2	1	–	–	1	1	–	1	1	–	–	–	–	–	1	1	–	–	–	–	–	–	–	–	–	–	15
14·0	–	–	–	–	–	–	–	–	–	–	2	–	1	1	1	1	–	–	1	3	1	1	4	2	–	1	–	1	1	1	–	1	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	23
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	1	1	–	1	3	7	1	4	4	4	2	3	–	–	1	–	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	36
2	–	–	–	–	1	–	–	–	1	1	–	1	–	–	1	1	1	2	2	3	5	3	7	–	4	4	1	5	1	5	2	3	1	3	–	–	–	–	1	–	–	–	1	–	–	–	–	–	60
3	–	–	–	–	–	–	–	–	–	1	–	1	1	–	2	2	1	1	3	8	4	5	6	6	8	4	8	2	2	4	1	2	2	1	–	–	–	1	–	–	–	–	–	–	–	1	–	–	–	77
4	–	–	–	–	–	–	–	–	–	–	1	–	1	–	2	3	3	5	1	6	4	8	6	10	6	4	7	4	3	8	3	2	5	–	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	95
5	–	–	–	–	–	–	–	–	–	1	1	–	–	–	–	3	3	2	7	4	5	8	5	6	14	8	9	8	13	12	10	6	10	4	4	1	3	–	1	1	–	1	–	–	–	–	–	–	–	150
6	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	3	6	4	4	8	8	13	6	10	10	14	13	20	13	11	7	9	5	5	9	4	3	2	2	2	1	–	–	–	–	–	–	–	–	193
7	–	–	–	–	–	–	–	–	–	–	2	–	1	1	–	2	4	9	6	6	1	12	12	13	15	15	15	16	18	7	13	8	10	5	4	2	2	1	1	–	1	–	–	–	–	–	–	–	–	202
8	–	–	–	–	–	–	–	–	–	–	–	–	–	1	3	–	1	4	6	5	12	9	11	11	11	12	17	15	26	13	9	12	7	6	5	2	3	–	–	–	–	–	2	–	–	–	–	–	–	203
9	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	3	–	3	6	2	8	7	12	12	10	19	17	25	18	16	22	10	11	5	8	1	2	4	–	1	1	–	–	–	–	–	–	–	–	223
15·0	–	–	–	–	–	–	–	–	1	–	–	–	–	–	2	3	5	5	3	3	8	12	10	11	21	12	13	19	17	18	14	8	10	6	9	7	5	2	–	2	1	–	1	–	–	–	–	–	–	–	228
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	3	–	3	2	5	11	8	11	5	17	13	27	26	18	14	21	9	2	10	8	5	4	7	1	–	2	–	2	–	–	–	–	–	234
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	3	2	7	12	6	10	9	16	18	16	16	19	18	8	14	11	7	5	6	4	2	1	4	3	–	2	–	1	–	–	–	–	221
3	1	–	–	–	–	1	–	–	–	–	–	–	–	1	1	–	2	2	–	3	4	6	8	2	8	10	19	16	14	19	10	12	19	10	14	6	7	3	3	–	6	3	3	–	–	–	–	–	–	213
4	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1	–	–	–	1	4	4	3	3	1	4	5	13	10	12	16	12	8	7	14	9	10	11	8	5	1	2	2	–	–	1	–	–	–	–	168
5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	2	3	3	4	7	10	13	12	10	7	5	7	4	9	8	6	7	8	6	7	3	3	1	1	–	–	1	–	–	–	153
6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	1	3	3	5	6	5	12	5	8	5	11	9	16	18	9	6	9	6	7	1	2	1	1	–	–	1	–	–	–	151
7	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	1	–	1	2	2	2	2	5	5	11	5	9	6	6	7	7	3	1	1	5	3	6	–	1	1	–	1	–	–	–	95
8	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	2	1	1	3	1	3	3	8	4	8	11	4	1	4	3	3	3	4	1	3	2	–	–	–	–	–	–	–	75
9	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	1	1	–	3	–	4	3	–	2	1	1	3	4	3	4	2	9	2	–	2	1	3	–	1	–	–	–	–	–	51
16·0	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1	–	–	1	1	4	2	1	–	4	2	5	3	1	2	3	3	1	1	–	–	–	–	–	–	–	36
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	2	–	–	–	4	2	1	1	2	1	–	1	2	4	2	1	–	–	1	–	–	–	–	–	–	–	25
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	1	–	2	–	3	–	2	3	1	3	2	–	–	1	1	–	1	–	1	–	–	–	–	–	22
3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	1	2	–	–	–	–	–	–	–	1	–	–	–	–	–	6
4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	2	–	–	–	1	1	2	2	–	–	–	–	–	–	–	–	–	–	9
5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	2	–	–	–	1	–	1	–	–	–	–	–	–	–	–	–	7
6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	1	1	1	–	–	–	1	–	–	–	–	–	–	–	–	1	3
7	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	1
8	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0
9	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
Totals	1	–	–	–	1	1	–	1	3	4	6	4	7	7	12	24	35	38	51	57	88	123	114	128	153	175	199	223	241	187	176	172	142	133	114	84	79	53	54	33	30	24	5	11	1	3	2	–	–	1	3000

TABLE II. 3000 *Criminals.* Height (feet and inches).

TABLE II. 3000 *Criminals*. Height (feet and inches).

Head Breadth (millimetres).	4′ 7″/16—8′ 9/16	8′ 9/16—9′ 9/16	9′ 9/16—10″ 9/16	10″ 9/16—11″ 9/16	5′ 11″ 9/16—0″ 9/16	0″ 9/16—1″ 9/16	1″ 9/16—2″ 9/16	2″ 9/16—3″ 9/16	3″ 9/16—4″ 9/16	4″ 9/16—5″ 9/16	5″ 9/16—6″ 9/16	6′ 7″ 9/16—8″ 9/16	8′ 9/16—9′ 9/16	9′ 9/16—10″ 9/16	10″ 9/16—11″ 9/16	6′ 11″ 9/16—0″ 9/16	0″ 9/16—1″ 9/16	1″ 9/16—2″ 9/16	2″ 9/16—3″ 9/16	3″ 9/16—4″ 9/16	4″ 9/16—5″ 9/16	Totals
13·4	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
5	—	—	—	1	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2
6	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
7	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	7
8	—	—	—	1	—	2	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	8
9	—	—	—	1	—	—	—	—	—	—	4	—	—	—	—	—	—	—	—	—	—	15
14·0	—	—	—	—	—	3	1	5	1	1	2	3	—	—	—	—	—	—	—	—	—	23
1	—	—	—	1	1	1	2	5	5	1	3	1	—	—	—	—	—	—	—	—	—	36
2	—	—	—	1	—	2	1	6	7	5	9	5	—	—	—	—	—	—	—	—	—	60
3	—	—	—	1	1	4	9	6	6	7	10	7	4	1	1	—	—	—	—	—	—	77
4	—	—	—	2	4	4	8	6	13	5	11	12	—	1	2	1	—	—	—	—	—	95
5	—	—	—	1	4	2	7	11	17	7	19	7	5	3	1	—	1	—	—	—	—	150
6	—	—	1	—	2	5	6	15	21	29	23	15	6	2	2	—	—	—	—	—	—	193
7	—	—	1	1	5	5	5	17	27	33	26	18	8	7	2	—	—	1	—	—	—	202
8	—	—	—	1	2	6	5	15	25	34	31	18	15	8	2	1	—	—	—	—	—	203
9	—	—	—	—	2	6	6	15	27	36	24	28	19	14	4	1	—	—	—	—	—	223
15·0	—	—	—	1	6	10	10	25	20	38	37	20	23	14	6	—	—	—	—	—	1	228
1	—	—	—	1	3	4	10	31	36	33	34	29	17	12	4	3	1	—	—	—	—	234
2	—	1	—	—	1	5	10	26	39	33	30	45	15	19	7	2	—	—	—	—	—	221
3	—	—	—	1	4	6	12	28	30	37	35	28	17	14	8	3	2	1	—	—	—	213
4	—	—	—	—	4	8	10	18	26	34	47	18	26	6	7	5	1	—	—	—	—	168
5	—	—	—	—	4	8	10	12	19	25	30	25	13	8	4	5	2	1	—	—	—	153
6	—	—	—	—	—	1	8	15	16	27	13	33	22	10	6	1	1	—	—	—	—	151
7	—	—	—	—	3	5	5	14	17	26	23	16	17	14	6	3	2	—	—	—	—	95
8	—	—	—	—	—	1	2	9	16	11	12	19	8	5	3	5	1	1	—	—	—	75
9	—	—	—	1	1	3	12	16	5	13	16	11	10	7	5	1	—	—	—	—	—	51
16·0	—	—	—	—	1	—	—	4	6	7	9	6	7	3	5	2	—	—	—	—	—	36
1	—	—	—	1	1	1	1	2	2	7	4	12	1	3	2	—	—	—	—	—	—	25
2	—	—	—	—	1	1	—	3	2	3	5	4	4	2	—	—	—	1	—	—	—	22
3	—	—	—	—	—	—	—	—	—	2	3	4	6	3	1	—	—	—	—	—	—	6
4	—	—	—	—	—	—	—	1	—	—	—	—	1	2	1	1	1	—	—	—	—	9
5	—	—	—	—	—	—	—	—	2	—	—	3	—	—	—	—	—	—	—	—	—	7
6	—	—	—	—	—	—	—	1	1	1	—	1	2	—	1	1	—	—	—	—	—	3
7	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
9	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
Totals	1	1	6	23	48	90	175	317	393	462	458	413	264	177	97	46	17	7	4	—	1	3000
Means	142	152	148·2	146·4	148·1	149·0	149·0	149·8	149·8	150·6	150·4	151·1	151·5	151·6	151·7	152·3	153·1	153·7	152·5	—	—	149

TABLE III. 3000 *Criminals.* Height (feet and inches).

Left Middle Finger (millimetres).	4′ 7″/16—8″ 9/16	8″ 9/16—9″ 9/16	9″ 9/16—10″ 9/16	10″ 9/16—11″ 9/16	5′ 11″ 9/16—0″ 9/16	0″ 9/16—1″ 9/16	1″ 9/16—2″ 9/16	2″ 9/16—3″ 9/16	3″ 9/16—4″ 9/16	4″ 9/16—5″ 9/16	5″ 9/16—6″ 9/16	6″ 9/16—7″ 9/16	7″ 9/16—8″ 9/16	8″ 9/16—9″ 9/16	9″ 9/16—10″ 9/16	10″ 9/16—11″ 9/16	6′ 11″ 9/16—0″ 9/16	0″ 9/16—1″ 9/16	1″ 9/16—2″ 9/16	2″ 9/16—3″ 9/16	3″ 9/16—4″ 9/16	4″ 9/16—5″ 9/16	Totals
9·4																							—
5						1																	1
6																							0
7																							0
8							1																1
9			1		1	1	1																3
10·0	1			1	3	3	2	1															7
1				1	2	2		1															7
2				2	2	2	3	2															10
3		1	2	3	2	2	3	5															17
4			1	1	1	3	3	4															20
5				1	3	2	6	6	3	1													30
6				1	4	4	9	14	6	3	1												44
7			1	2	4	5	14	16	15	7	3	1											74
8				2	5	6	14	27	10	7	1	2											75
9				2	2	6	14	24	27	14	10	4											102
11·0					2	6	15	31	37	27	17	10	6										163
1					3	3	22	26	24	26	24	7	4										152
2					3	3	21	30	38	29	27	20	4	1									183
3					1	2	10	24	26	39	26	24	7	2									164
4						3	5	9	26	58	56	22	10	11									228
5							5	4	9	56	33	26	34	25	11	2							233
6							6	6	5	33	37	38	38	27	12	2							226
7							12	12	11	37	39	48	45	24	9	2							232
8							7	9	24	30	37	48	48	29	10	2							184
9							5	11	15	35	41	34	35	19	10	8	2						162
12·0							6	17	30	35	27	32	35	22	16	8	2	1					163
1							2	12	10	27	19	42	39	22	27	10	4	1					126
2							1	4	8	19	13	22	28	15	27	16	4	1					91
3								2	4	13	6	23	17	16	11	8	1						89
4							1	2	5	6	8	10	13	20	23	6	5						44
5								1	4	8	2	7	12	7	7	7	1						52
6							1	1	1	2	1	3	12	12	11	6	8	2	1				35
7											1		3	5	7	8	6	3	1				31
8											1	1	7	5	5	8	2	2					25
9												1	2	3	1	8	5	1	1				7
18·0												1	2	2	2	8	3	2					8
1														1	—	—	1	—					2
2													1	1									6
3																	1		1				2
4																		1					2
5																							1
Totals	1	1	6	23	48	90	175	317	393	402	458	413	264	177	97	46	17	7	4	0	0	1	3000
Means	100	103	102·8	107·0	107·8	109·4	110·6	111·8	115·3	114·8	116·5	117·7	118·6	120·1	122·2	123·9	125·9	126·4	127·7	—	—	112	

TABLE IV. 1306 *Criminals.*

Head Length (millimetres).

Head Breadth (millimetres).

	17·									18·										19·										20·									21·	
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*0*	Totals
13·4	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
5	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
6	–	–	–	–	–	–	–	1	1	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	4
7	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	3
8	–	–	–	–	–	–	–	–	–	–	3	–	–	–	1	–	–	–	–	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	6
9	1	–	–	–	–	–	1	–	–	–	–	–	–	1	–	1	1	–	–	–	–	–	–	1	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	7
14·0	–	–	–	–	2	–	–	1	–	–	–	–	3	1	–	1	–	–	–	–	1	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	9
1	–	–	–	–	–	–	–	1	–	–	1	1	–	–	2	5	–	3	2	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	18
2	–	–	1	–	1	–	–	–	1	1	1	1	–	2	–	4	–	1	1	1	1	1	2	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	21
3	–	–	–	–	–	1	1	–	2	–	–	1	1	5	2	2	2	4	4	1	4	1	–	3	1	2	–	–	–	–	1	–	–	–	–	–	–	–	1	–	39
4	–	–	1	–	–	–	–	–	–	1	1	2	–	3	1	3	3	6	4	4	3	3	2	3	3	–	3	–	–	–	–	–	–	–	–	–	–	–	–	–	46
5	–	–	–	–	–	–	–	–	2	1	–	7	1	2	6	2	3	5	4	5	3	5	4	6	1	3	1	2	1	1	–	–	–	–	–	–	–	–	–	–	65
6	–	–	–	–	–	–	–	–	2	1	1	1	3	3	5	4	6	7	10	7	7	6	5	3	2	4	2	4	2	1	1	1	1	1	–	–	–	–	–	–	90
7	–	–	–	–	–	1	1	–	1	1	2	1	2	–	3	10	5	6	8	6	8	8	5	4	3	6	2	3	2	1	1	1	–	1	–	–	–	–	–	–	92
8	–	–	–	–	–	–	–	2	–	–	3	3	3	4	4	5	4	5	3	4	7	11	6	3	4	6	4	2	–	2	–	–	–	–	–	–	1	–	–	–	86
9	–	–	–	–	–	–	–	–	–	3	–	2	3	1	5	2	3	7	4	7	8	12	7	10	6	4	3	2	3	1	–	3	–	1	1	–	–	–	–	–	98
15·0	–	1	–	–	–	–	–	1	2	2	3	2	1	5	6	4	4	5	8	5	5	5	6	8	3	2	3	5	1	2	1	–	1	–	–	1	–	–	–	–	92
1	–	–	–	–	–	–	–	–	–	–	–	2	2	–	5	5	4	2	11	6	11	10	10	10	8	2	1	3	4	2	2	4	–	–	–	–	–	–	–	–	104
2	–	–	–	–	–	–	–	–	–	–	1	2	–	6	7	3	5	3	6	9	7	6	7	9	3	6	5	1	1	3	2	–	1	2	1	–	1	–	–	–	97
3	–	–	–	–	–	–	1	–	1	2	–	–	3	3	3	–	4	5	11	5	6	7	4	5	8	4	7	1	3	1	1	–	4	1	1	–	–	–	–	–	91
4	–	–	–	–	–	–	1	1	–	–	–	1	1	2	1	1	–	1	–	7	4	7	8	6	5	4	6	2	5	2	3	4	–	1	–	–	–	–	–	–	73
5	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	2	4	1	2	5	5	5	2	2	2	3	3	3	3	3	3	2	3	1	–	–	–	–	–	–	56
6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	4	3	–	4	4	2	2	5	5	6	7	4	3	3	1	–	–	–	1	–	–	–	–	–	57
7	–	–	–	–	–	–	–	–	–	1	–	1	1	–	–	–	1	2	2	3	2	5	3	3	3	2	1	1	1	1	1	4	1	4	–	1	1	–	1	–	46
8	–	–	–	–	–	–	–	–	–	–	–	1	–	1	1	–	–	2	1	2	3	2	–	1	1	3	1	2	1	2	–	1	1	–	1	–	–	–	–	–	27
9	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	2	–	1	–	–	2	–	–	3	2	2	2	–	3	–	–	1	–	2	–	1	–	–	–	22
16·0	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	1	1	2	1	1	–	3	–	3	–	1	1	–	1	1	1	–	–	–	–	–	18
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	2	1	–	1	1	–	–	1	1	1	–	–	–	1	–	–	–	–	–	10
2	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	1	–	1	–	–	–	1	2	1	2	2	–	–	–	–	–	–	1	–	–	–	–	12
3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	1	1	–	–	–	–	–	–	–	–	–	1	–	–	–	5
4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	–	–	–	–	1	–	1	1	–	–	–	–	–	–	–	–	–	5
5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	3
6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	1
7	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0
8	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0
9	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
Totals	1	1	2	0	3	3	5	7	12	13	17	28	24	43	53	57	55	68	83	85	96	102	79	83	66	66	56	43	35	30	20	24	14	13	8	3	6	0	1	1	1306

TABLE V. 1306 *Criminals.*

Height (feet and inches).

Head Breadth (millimetres).	4′ 7″/16 — 8″ 9/16	8″ 9/16 — 9″ 9/16	9″ 9/16 — 10″ 9/16	10″ 9/16 — 11″ 9/16	5′ 11″ 9/16 — 0″ 9/16	0″ 9/16 — 1″ 9/16	1″ 9/16 — 2″ 9/16	2″ 9/16 — 3″ 9/16	3″ 9/16 — 4″ 9/16	4″ 9/16 — 5″ 9/16	5″ 9/16 — 6″ 9/16	6″ 9/16 — 7″ 9/16	7″ 9/16 — 8″ 9/16	8″ 9/16 — 9″ 9/16	9″ 9/16 — 10″ 9/16	10″ 9/16 — 11″ 9/16	6′ 11″ 9/16 — 0″ 9/16	0″ 9/16 — 1″ 9/16	1″ 9/16 — 2″ 9/16	2″ 9/16 — 3″ 9/16	3″ 9/16 — 4″ 9/16	4″ 9/16 — 5″ 9/16	Totals
13·4	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
5	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
6	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	4
7	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	3
8	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	6
9	—	—	—	—	2	3	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	7
14·0	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	9
1	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	18
2	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	21
3	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	39
4	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	46
5	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	65
6	—	—	—	—	2	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	90
7	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	92
8	—	—	—	—	2	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	86
9	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	98
15·0	—	—	—	—	2	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	92
1	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	104
2	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	97
3	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	91
4	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	73
5	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	56
6	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	57
7	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	46
8	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	27
9	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	22
16·0	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	18
1	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	10
2	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	12
3	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	5
4	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	5
5	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	3
6	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	1
7	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	0
8	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	0
9	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	1
Totals	—	1	—	15	23	48	81	151	171	193	193	175	103	80	37	26	3	4	1	—	—	1	1306

TABLE VI. 1306 *Criminals.* Height (feet and inches).

TABLE VI. 1306 *Criminals.* Height (feet and inches).

Left Middle Finger (millimetres).

	4′ 7″ 9/16—8″ 9/16	8″ 9/16—9″ 9/16	9″ 9/16—10″ 9/16	10″ 9/16—11″ 9/16	5′ 11″ 9/16—0″ 9/16	0″ 9/16—1″ 9/16	1″ 9/16—2″ 9/16	2″ 9/16—3″ 9/16	3″ 9/16—4″ 9/16	4″ 9/16—5″ 9/16	5″ 9/16—6″ 9/16	6″ 9/16—7″ 9/16	7″ 9/16—8″ 9/16	8″ 9/16—9″ 9/16	9″ 9/16—10″ 9/16	10″ 9/16—11″ 9/16	6′ 11″ 9/16—0″ 9/16	0″ 9/16—1″ 9/16	1″ 9/16—2″ 9/16	2″ 9/16—3″ 9/16	3″ 9/16—4″ 9/16	4″ 9/16—5″ 9/16	Totals
9·4																							0
5																							1
6						1																	0
7																							0
8																							1
9					1																		1
10·0						1	1																4
1					1	2	1	1															1
2				1	1	2	2	3															5
3				1	1	1	2	9	1														10
4				1	1	2	2	7	3														10
5					1	1	4	16	6														13
6					1	2	3	7	3														13
7					2	2	2	12	6														22
8					2	2	2	12	1														21
9					2	4	7	8	2														12
11·0					4	7	5	13	1														12
1						5	11	10															7
2					1	10	12	8															3
3						4	12	11															4
4						9	13	13															0
5						3	12	12															3
6						2	9	17															0
7							7	12															0
8							3	18															1
9							4	16															1
12·0							1	16															1
1							2	12															1
2							1	12															1
3							1	9															1
4							1	6															1
5								6															1
6								4															1
7								3															1
8								1															1
9								1															1
13·0								1															1
1								1															1
2								1															1
3								1															1
4								1															1
5								1															1
Totals	—	1	—	15	23	48	81	151	171	193	193	175	103	80	37	26	3	4	1	—	—	1	1306

TABLE VII.

1000 *Cambridge Men.*

Head Length (ins.)

Head Breadth (ins.)	6·8	6·9	7	7·1	7·2	7·3	7·4	7·5	7·6	7·7	7·8	7·9	8	8·1	8·2	8·3	8·4	8·5	8·6	Totals
5·5	—	—	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	3
5·6	—	—	—	—	2	3	1	4	2	—	—	—	—	—	—	—	—	—	—	12
5·7	1	1	—	1	2	6	5	7	5	8	3	4	—	—	—	—	—	—	—	43
5·8	—	—	2	4	6	7	15	12	12	11	7	3	1	—	—	—	—	—	—	80
5·9	—	—	1	4	6	16	16	24	20	28	7	5	4	—	—	—	—	—	—	131
6	—	1	—	6	14	12	24	40	45	40	28	9	13	4	—	—	—	—	—	236
6·1	—	—	1	3	4	6	11	27	42	40	22	15	10	2	2	—	—	—	—	185
6·2	—	—	1	2	6	3	14	18	22	25	13	14	17	2	4	1	—	—	—	142
6·3	—	—	1	—	1	4	9	8	16	15	15	16	6	7	—	—	—	—	1	99
6·4	—	—	—	—	1	—	2	4	6	6	3	4	7	2	1	1	—	—	—	37
6·5	—	—	—	—	—	1	1	—	3	3	—	2	3	1	1	—	—	—	—	15
6·6	—	—	—	1	—	—	—	1	2	3	—	1	—	2	1	1	—	—	—	12
6·7	—	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	—	3
6·8	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
Totals	1	2	6	21	43	58	100	145	175	180	98	75	61	22	9	3	—	—	1	1000

TABLE VIII.

1000 *Cambridge Men.*

Height (feet and inches).

Head Breadth (ins.)	5′ 1½–2½	2½–3½	3½–4½	4½–5½	5½–6½	6½–7½	7½–8½	8½–9½	9½–10½	10½–11½	11½–6′0½	6′ 0½–1½	1½–2½	2½–3½	3½–4½	4½–5½	Totals
5·5	—	—	—	—	—	1	1	—	—	0·5	0·5	—	—	—	—	—	3
5·6	0·5	0·5	—	1	1	3	1·5	1·5	0·5	1	1·5	—	—	—	—	—	12
5·7	1	—	—	—	2	5·5	6·5	9	10	5	2·	1	—	—	1	—	43
5·8	—	3	4	6	11	13·5	11	12	9·5	5·5	3·5	—	—	—	1	—	80
5·9	2	4	3·5	4·5	14	16	19	20·5	22	8·5	12·5	2	1·5	1	—	—	131
6·	—	4	6	5·5	22·5	32·5	42·5	38	26	28·5	8·5	14	6	1	1	—	236
6·1	—	3	5	9	12	27	22·5	35	24·5	23·5	8·5	4·5	8	2·5	—	—	185
6·2	0·5	3·5	4	6·5	11·5	12·5	15·5	28	24·5	16	5·5	7	2	3	1·5	0·5	142
6·3	—	—	2	7	5	11·5	9·5	22	12·5	13·5	4·5	8·5	—	2·5	0·5	—	99
6·4	—	1	—	—	3	0·5	5	8	4	4	4·5	6	1	—	—	—	37
6·5	—	—	—	—	—	—	1	4	3	1	1	2·5	0·5	2	—	—	15
6·6	—	—	—	1	1·5	0·5	4	1	—	1	1	2	—	—	—	—	12
6·7	—	—	—	—	1	—	—	—	—	—	—	—	2	—	—	—	3
6·8	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
Totals	4	19	24·5	40·5	84·5	123·5	139	179	138·5	108	53·5	47·5	21	12	5	0·5	1000

TABLE IX.

1000 *Cambridge Men.*

Head Length (inches).

Height (feet and inches).	6·8	6·9	7	7·1	7·2	7·3	7·4	7·5	7·6	7·7	7·8	7·9	8	8·1	8·2	8·3	8·4	8·5	8·6	Totals
5′ 1½—2½	—	1	—	—	0·5	—	—	2	0·5	—	—	—	—	—	—	—	—	—	—	4
2½—3½	—	—	1	2	2·5	2	2	4·5	2·	2	1	—	—	—	—	—	—	—	—	19
3½—4½	—	—	—	1	2	—	5·5	6	5·5	4·5	—	—	—	—	—	—	—	—	—	24·5
4½—5½	—	—	0·5	1	3	4	3	6·5	4·5	4·5	6·5	7	—	—	—	—	—	—	—	40·5
5½—6½	1	—	2·5	5·5	1	6	10·5	14	21	11	7	1·5	2	1	0·5	—	—	—	—	84·5
6½—7½	—	—	—	1·5	8·5	10	14·5	20	21·5	19·5	12	7·5	6	1	1·5	—	—	—	—	123·5
7½—8½	—	1	—	2·5	8·5	6·5	22·5	24·5	29	24	9·5	4	3	3	1	—	—	—	—	139
8½—9½	—	—	0·5	5·5	4	8·5	16·5	24·5	31·5	36·5	14	21·5	12	3	1	—	—	—	—	179
9½—10½	—	—	0·5	2	4	8·5	15	13·5	21·5	26	14·5	13·5	13	4	1·5	1	—	—	—	138·5
10½—11½	—	—	—	—	3	7·5	3·5	12	22·5	25·5	16	10	6·5	1	0·5	—	—	—	—	108
11½—6′ 0½	—	—	—	—	3	4	5	6·5	8·5	9	5·5	3	5	3	1	—	—	—	—	53·5
6′ 0½—1½	—	—	1	—	2	1	2	6	2	8·5	7·5	5	5	3	1·5	2	—	—	1	47·5
1½—2½	—	—	—	—	—	—	—	3	2	4·5	2·5	1	5·5	2	0·5	—	—	—	—	21
2½—3½	—	—	—	—	1	—	—	—	2	4·5	1	—	3	0·5	—	—	—	—	—	12
3½—4½	—	—	—	—	—	—	—	1·5	1	—	1	1	—	0·5	—	—	—	—	—	5
4½—5½	—	—	—	—	—	—	—	0·5	—	—	—	—	—	—	—	—	—	—	—	0·5
Totals	1	2	6	21	43	58	100	145	175	180	98	75	61	22	9	3	—	—	1	1000

TABLE XI.

Height (ins.)

Head Breadth (cms.)	under $64\frac{9}{16}$	$64\frac{9}{16}$ to $66\frac{9}{16}$	over $66\frac{9}{16}$	Totals
14·8 and under	455	323	299	1077
14·9 to *15·2* ...	316	277	313	906
over *15·2* ...	283	320	414	1017
Totals	1054	920	1026	3000

TABLE XII.

Height (ins.)

L. M. Finger (cms.)	under $64\frac{9}{16}$	over $64\frac{9}{16}$	Totals
11·5 and under	849	665	1514
over *11·5* ...	205	1281	1486
Totals	1054	1946	3000

TABLE XIII.

Height (ins.)

L. M. Finger (cms.)	under $64\frac{9}{16}$	$64\frac{9}{16}$ to $66\frac{9}{16}$	over $66\frac{9}{16}$	Totals
11·1 and under	515	149	42	706
11·2 to *11·9* ...	504	607	501	1612
over *11·9* ...	35	164	483	682
Totals	1054	920	1026	3000

TABLE XIV.

Head Length (cms.)

Head Breadth (cms.)	*19·1* and under	over *19·1*	Totals
14·8 and under	705	372	1077
over *14·8* ...	750	1173	1923
Totals	1455	1545	3000

TABLE XV.

Head Length (cms.)

Head Breadth (cms.)	*18·5* and under	*18·6* to *19·7*	over *19·7*	Totals
14·8 and under	279	728	70	1077
14·9 to *15·2* ...	113	670	123	906
over *15·2* ...	71	645	301	1017
Totals	463	2043	494	3000

TABLE XVI.

Head Length (cms.)

Face Breadth (cms.)		*19·1* and under	over *19·1*	Totals
	13·6 and under	921	579	1500
	over *13·6* ...	534	966	1500
	Totals	1455	1545	3000

TABLE XVII.

Head Length (cms.)

Face Breadth (cms.)		*18·8* and under	*18·9* to *19·3*	over *19·3*	Totals
	13·4 and under	453	357	229	1039
	13·5 to *13·8* ...	259	335	328	922
	over *13·8* ...	146	333	560	1039
	Totals	858	1025	1117	3000

TABLE XVIII.

Head Length (cms.)

Left Cubit (cms.)		*19·1* and under	over *19·1*	Totals
	44·9 and under	836	583	1419
	over *44·9* ...	619	962	1581
	Totals	1455	1545	3000

TABLE XIX.

Head Length (cms.)

Left Cubit (cms.)		*18·8* and under	*18·9* to *19·3*	over *19·3*	Totals
	44 and under	375	310	226	911
	44·1 to *45·7*	264	365	373	1002
	over *45·7* ...	219	350	518	1087
	Totals	858	1025	1117	3000

TABLE XX.

Head Length (cms.)

Left Foot (cms.)		*19·1* and under	over *19·1*	Totals
	25·5 and under	832	545	1377
	over *25·5* ...	623	1000	1623
	Totals	1455	1545	3000

TABLE XXI.

Head Length (cms.)

Left Foot (cms.)	*18·8* and under	*18·9* to *19·3*	over *19·3*	Totals
25 and under	378	300	205	883
25·1 to *26* ...	268	345	366	979
over *26* ...	212	380	546	1138
Totals	858	1025	1117	3000

TABLE XXII.

Head Length (cms.)

L. M. Finger (cms.)	*19·1* and under	over *19·1*	Totals
11·5 and under	880	634	1514
over *11·5* ...	575	911	1486
Totals	1455	1545	3000

TABLE XXIII.

Height (ins.)

Head Length (cms.)	$64\frac{9}{16}$ and under	over $64\frac{9}{16}$	Totals
19·1 and under	665	790	1455
over *19·1* ...	389	1156	1545
Totals	1054	1946	3000

TABLE XXIV.

Head Breadth (cms.)

Face Breadth (cms.)	*14·8* and under	over *14·8*	Totals
13·6 and under	834	666	1500
over *13·6* ...	243	1257	1500
Totals	1077	1923	3000

TABLE XXIV[a].

Head Breadth (cms.)

Face Breadth (cms.)	*15* and under	over *15*	Totals
13·6 and under	1077	423	1500
over *13·6* ...	451	1049	1500
Totals	1528	1472	3000

TABLE XXV.

Head Breadth (cms.)

L. M. Finger (cms.)	*14·8* and under	over *14·8*	Totals
11·5 and under	611	903	1514
over *11·5* ...	466	1020	1486
Totals	1077	1923	3000

TABLE XXV[a].

Head Breadth (cms.)

L. M. Finger (cms.)	*15* and under	over *15*	Totals
11·5 and under	841	673	1514
over *11·5* ...	687	799	1486
Totals	1528	1472	3000

TABLE XXVI.

Head Breadth (cms.)

Left Cubit (cms.)	*14·8* and under	over *14·8*	Totals
44·9 and under	570	849	1419
over *44·9* ...	507	1074	1581
Totals	1077	1923	3000

TABLE XXVII.

Left Foot (cms.)

Head Breadth (cms.)	*25·5* and under	over *25·5*	Totals
14·8 and under	587	490	1077
over *14·8* ...	790	1133	1923
Totals	1377	1623	3000

TABLE XXVIII.

Face Breadth (cms.)

L. M. Finger (cms.)	*13·6* and under	over *13·6*	Totals
11·5 and under	913	601	1514
over *11·5* ...	587	899	1486
Totals	1500	1500	3000

TABLE XXIX.

Face Breadth (cms.)

Left Cubit (cms.)	*13·6* and under	over *13·6*	Totals
44·9 and under	849	570	1419
over *44·9* ...	651	930	1581
Totals	1500	1500	3000

TABLE XXX.

Face Breadth (cms.)

Left Foot (cms.)	*13·6* and under	over *13·6*	Totals
25·5 and under	865	512	1377
over *25·5* ...	635	988	1623
Totals	1500	1500	3000

TABLE XXXI.

Face Breadth (cms.)

Height (ins.)	*13·6* and under	over *13·6*	Totals
$64\frac{9}{16}$ and under	683	371	1054
over $64\frac{9}{16}$ ...	817	1129	1946
Totals	1500	1500	3000

TABLE XXXII.

Left Foot (cms.)

L. M. Finger (cms.)	*25·5* and under	over *25·5*	Totals
11·5 and under	1103	411	1514
over *11·5* ...	274	1212	1486
Totals	1377	1623	3000

TABLE XXXIII.

L. M. Finger (cms.)

Left Cubit (cms.)	*11·5* and under	over *11·5*	Totals
44·9 and under	1196	223	1419
over *44·9* ...	318	1263	1581
Totals	1514	1486	3000

TABLE XXXIV.

Left Foot (cms.)

Left Cubit (cms.)	25·5 and under	over 25·5	Totals
44·9 and under	1089	330	1419
over 44·9 ...	288	1293	1581
Totals	1377	1623	3000

TABLE XXXV.

Height (ins.)

Left Cubit (cms.)	$65\frac{9}{16}$ and under	over $65\frac{9}{16}$	Totals
44·9 and under	1158	261	1419
over 44·9 ...	358	1223	1581
Totals	1516	1484	3000

TABLE XXXV[a].

Height (ins.)

Left Cubit (cms.)	$64\frac{9}{16}$ and under	over $64\frac{9}{16}$	Totals
44·9 and under	902	517	1419
over 44·9 ...	152	1429	1581
Totals	1054	1946	3000

TABLE XXXVI.

Height (ins.)

Left Foot (cms.)	$65\frac{9}{16}$ and under	over $65\frac{9}{16}$	Totals
25·5 and under	1088	289	1377
over 25·5 ...	428	1195	1623
Totals	1516	1484	3000

TABLE XXXVI[a].

Height (ins.)

Left Foot (cms.)	$64\frac{9}{16}$ and under	over $64\frac{9}{16}$	Totals
25·5 and under	873	504	1377
over 25·5 ...	181	1442	1623
Totals	1054	1946	3000

MENDEL'S LAWS OF ALTERNATIVE INHERITANCE IN PEAS.

By W. F. R. WELDON.

[Received December 9, 1901.]

If two plants, or two animals, of different characters be allowed to breed together, the parental characters may affect the offspring in any one of three ways. In the most usual case, the characters by which the parents differ may appear so intimately blended in the offspring that each young animal or plant appears intermediate in character between its parents, but it is not generally possible for us at present to resolve its body into separate elements, some of which resemble one parent, and some the other. In other cases, however, the body of the young is easily divisible into regions, in some of which the character of one parent is presented in a recognisable and often apparently unaltered state, while the rest of the body presents a similar resemblance to the other parent. In the cases of the third class the body of the offspring may entirely resemble that of one parent, the characters of the other being apparently unrepresented. While it is perfectly possible and indeed probable that the difference between these three forms of inheritance is only one of degree, it is still convenient to discuss them separately.

The work of Galton (No. 13) and Pearson (No. 24) has given us an expression for the effects of *blended* inheritance which seems likely to prove generally applicable, although the constants of the equations which express the relation between divergence from the mean in one generation and that in another may require modification in special cases. Our knowledge of *particulate* or mosaic inheritance, and of *alternative* inheritance, is however still rudimentary, and there is so much contradiction between the results obtained by different observers, that the evidence available is difficult to appreciate. It is the purpose of this essay to describe some cases of alternative inheritance, which have lately excited attention, and to discuss the contradictory evidence concerning them.

I.—*Mendel's Results with Peas.*

In 1865 Gregor Mendel (No. 21) described the results of crossing various races of the common Pea. He does not waste time in discussing the question whether all his races belong to one "species" or not, but describes the result of crossing any two of them as "hybrid." It is probable, however, that all his races belonged to what is now called *Pisum sativum*, to the exclusion of *Pisum arvense*.

The races used differed, no doubt, in many ways: but special attention is paid to seven sets of characters, with regard to each of which it was possible to separate the races into two categories. Thus the shape of the seeds might be round, with only slight and shallow wrinkles on the surface, or irregular and deeply wrinkled. The cotyledons of the seeds might be yellow or green in colour, and so on. The pairs of characters, recognised in this way for each organ or set of organs studied, are distinguished, according to their power of affecting hybrid offspring, into *dominant* and *recessive*. The characters presented by a race are not necessarily all "dominant," or all "recessive"; thus the character of roundness in seeds is dominant, that of wrinkled irregular shape recessive; yellow colour of cotyledons is dominant, and green recessive; but a race of peas may have smooth and rounded seeds with green cotyledons, or yellow cotyledons and wrinkled seeds.

The first general result obtained by Mendel may be stated as follows: *If peas of two races be crossed, the hybrid offspring will exhibit only the dominant characters of the parents; and it will exhibit these without (or almost without) alteration, the recessive characters being altogether absent, or present in so slight a degree that they escape notice.*

This may be called the Law of Dominance, and it at once explains the terms dominant and recessive.

The second result is that: *If the hybrids of the first generation, produced by crossing two races of peas which differ in certain characters, be allowed to fertilise themselves, all possible combinations of the ancestral race-characters will appear in the second generation with equal frequency, and these combinations will obey the Law of Dominance, so that characters intermediate between those of the ancestral races will not occur.*

From its consequences, this may be called the Law of Segregation.

The significance of these results may most easily be seen by considering an example. It has been said that yellowness of cotyledons is dominant over greenness of cotyledons. That is to say, if a plant from a race with yellow cotyledons and one from a race with green cotyledons be cross-fertilised, the resulting seeds will have yellow cotyledons, no matter which plant be used as the female parent. Mendel chose ten plants, some from a race with green and some from a race with yellow cotyledons, and upon these plants he made 58 crosses, so that the ♀ parent was sometimes of green-seeded, sometimes of yellow-seeded race. The resulting seeds had always yellow cotyledons. From these seeds

258 plants were grown, which produced 8023 seeds; and these seeds had the characters indicated by the law of segregation. The combinations of pure ancestral character, which are here possible, are four; since the gamete of either sex may conceivably be capable of transmitting either "yellowness" or "greenness" of cotyledon; Mendel's assumption is that every gamete inherits only one of these "alternative" characters, half the gametes of either sex inheriting each character, or the power of transmitting it, so that there may conceivably be the following combinations:

♀ yellow × yellow ♂
♀ yellow × green ♂
♀ green × yellow ♂
♀ green × green ♂.

But by the law of dominance, if unlike gametes unite, the resultant seed has the dominant character, while if similar gametes unite the resultant seed has naturally the character transmitted by both its constituent gametes. It follows that three of the above combinations give rise to seeds with yellow cotyledons, and only one to green seeds; and if each combination occurs equally often, the chance that a given hybrid seed of the second generation will be yellow is $\frac{3}{4}$, the chance that it will be green is $\frac{1}{4}$.

Now, of the 8023 seeds of Mendel's second generation, 6022 were yellow and 2001 green. Seeds of intermediate colour did not occur. The ratio between either of these numbers and the number of seeds observed is an excellent approximation to that required by Mendel's law of segregation*. The plants of the first hybrid generation invariably bore seeds of both colours, and as a rule seeds of both colours were associated in the same pod. Pods containing only yellow seeds did occur, pods with green seeds only did not. This is all in accordance with the law of segregation; for the number of peas per pod being from six to nine, the chance of getting a pod with yellow seeds only is from $(\frac{3}{4})^6$ for pods with six seeds to $(\frac{3}{4})^9$ for pods with nine seeds, or from 0·18 to 0·075; so that about 18 per cent. of smaller, and about 8 per cent. of larger pods, should contain only yellow seeds; but the chance of a pod containing even six green and no yellow seeds is only $(\frac{1}{4})^6 = \frac{1}{4096}$. Assuming that there were some 1500 pods on the plants, it is clear that the absence of pods without yellow seeds is in good accord with Mendel's law of dominance.

The third hybrid generation must, if the foregoing statements are true, be heterogeneous, for the hybrid seeds of the second generation are said to be of three kinds; those formed by the union of two gametes each transmitting only dominant characters, those formed from gametes each transmitting only recessive characters, and those formed by the union of dissimilar gametes. Therefore the offspring of these seeds should also be of three kinds; in the case before us, one kind, produced from yellow seeds, should give rise to yellow-seeded plants, like

* See below, Table I. p. 233.

those of their pure-bred yellow-seeded ancestors; one kind, produced from green seeds, should give rise to apparently pure-bred green-seeded plants; while the third kind, being yellow seeds produced by the union of one gamete with "dominant" and one with "recessive" properties, should give rise to plants identical with the hybrids of the first generation. This result was actually obtained. Those seeds of the second hybrid generation, which had green cotyledons, gave rise to plants which in turn produced only green seeds. From the yellow seeds of the second hybrid generation 519 plants were raised, of which 166 produced only yellow seeds, behaving therefore like pure-bred plants of yellow race, and 353 had both yellow and green seeds, in the proportion of three yellow to one green, as in the first hybrid generation. The observations were continued through six generations; and the descendants of those plants of the second generation which produced only one kind of seed remained throughout apparently pure-bred, producing each its proper kind of seed as regularly as its pure-bred ancestors; the hybrids of every generation behaved exactly like those of the first or second generation. The result of this process is that the percentage of hybrids diminishes in every generation: for suppose each plant in each generation to produce four seeds only, the plants of the first hybrid generation will produce one apparently pure-bred yellow-seeded and one apparently pure-bred green-seeded plant, with two hybrids; in the third generation there will be four apparently pure-bred yellow plants from the apparently pure yellow-seeded plant of the second generation, and two others, one from each hybrid; there will in the same way be six apparently pure green-seeded plants, and four hybrids. Assuming this process to go on, the numbers in successive generations will be:

Generation	Yellow	Hybrid	Green
1		1	
2	1	2	1
3	6	4	6
4	28	8	28
5	120	16	120
6	496	32	496
etc.			

So that the number of apparently pure-bred forms of each kind increases, and in the $(n+1)$th hybrid generation, the three classes of plants are in the proportions of $2^n-1 : 2 : 2^n-1$.

Results closely similar to these were obtained with seven sets of differential characters; namely

1. *The shape of the seed* (round and feebly wrinkled, *dominant*; irregular and deeply wrinkled, *recessive*).

2. *The colour of the cotyledons*, already dealt with.

3. *The colour of the seed-coat* (grey, grey-brown, or leather-coloured, with or without violet dots, associated with purple-violet flowers and red colouring round leaf-axils, *dominant*; white, associated with white flowers, *recessive*).

4. *The shape of the ripe pod* (not constricted, smooth, *dominant*; constricted between the seeds, *recessive*).

5. *The colour of the unripe pod* (green, associated with green stem, mid-ribs of leaves, and calyx, *dominant*; yellow, associated with similar colouring of stem, mid-ribs and calyx, *recessive*).

6. *The distribution of the flowers* (scattered along the axis, *dominant*; gathered into a short umbel-like cluster, at the extremity of the axis, *recessive*).

7. *The length of the main stem* (tall, *dominant*; short, *recessive*).

The only qualifications Mendel offers, in applying his general statements to these very varied characters, are (1) that the violet dots on the seed-coat are often more numerous and larger in hybrids than in pure-bred forms, and (2) the observation that the mere fact of hybridisation produces an increase in the size of the vegetative organs, so that hybrid plants are often taller than either of their parents, an observation made previously by Knight (No. 18) for peas, and by many later naturalists for peas and other plants (see the summary of the evidence by Darwin, No. 9, Vol. II. Chap. 17, and in addition to the authorities there cited, Naudin, No. 22. For extensive observations on Peas, see Tschermak, Nos. 27 and 28).

It is clearly important to test these remarkable statements by a careful study of the numerical results, and by the application of such tests as may be possible. It seems to me that by neglecting these precautions some writers have been led to overlook the wonderfully consistent way in which Mendel's results agree with his theory, saying that his numbers "are not large enough to give really smooth results," and while they thus unwillingly do rather less than justice to Mendel's own work, at the same time they accept results which seem really inconsistent with those obtained by Mendel as proof that his statements are applicable to a wider range of cases than those he actually observed.

Mendel's observations fall into two groups; the first group relates to a series of cross-fertilisations, in which each pair of races crossed differed in only one of the seven sets of characters dealt with; the second group contains observations on races which differed in two or more sets of characters.

The observations of the first group are more numerous than those of the second, and may be considered first.

The seven sets of observations, showing the dominance of one character in the first hybrid generation, must of course rest upon Mendel's statement, which I think no one who reads his paper will find the slightest difficulty in accepting. The behaviour of hybrids of the second generation can be tested from the numbers

which are given. From what has been said concerning yellow and green cotyledons, it is evident that the chance of getting a dominant character in a plant of the second hybrid generation is the chance that one or other of three combinations, out of four which are all possible and equally probable, occurred during the fusion of the gametes which gave rise to the plant. The chance that a given plant of the second hybrid generation will present dominant characters is therefore $\frac{3}{4}$; and the records may be treated as attempts to verify this experimentally. Now if a series of n things be observed, and the chance that any one of them has a particular character is $\frac{3}{4}$, the "probable error" of the expectation that $\frac{3}{4}n$ of the things observed will show this character is well known to be $0{\cdot}67449\sqrt{n \times \frac{3}{4} \times \frac{1}{4}}$; so that if a number of such series are observed, in half of them the frequency with which the desired character occurs will lie between the limits $\frac{3}{4}n \pm 0{\cdot}67449\sqrt{n \times \frac{3}{4} \times \frac{1}{4}}$, and in half of them it will lie outside these limits. In each of Mendel's records of a second hybrid generation the probable error has been calculated, and the result is shown in the following table:

TABLE I.

Individuals with Dominant Characters in the Second Hybrid Generation.

Characters Crossed	Individuals of Second Hybrid Generation	Number of Dominant Individuals	Dominant Individuals on Mendel's Theory	Probable Error of Theory	Deviation of Observation from Theory
1. (Shape of Seeds) ...	7324	5474	5493	±24·995	−19
2. (Colour of Cotyledons)...	8023	6022	6017·25	±26·160	+ 4·75
3. (Colour of Seed Coats)...	929	705	696·75	± 8·902	+ 8·25
4. (Shape of Pod)	1181	882	885·75	±10·037	− 3·75
5. (Colour of Pod)... ...	580	428	435	± 7·034	− 7
6. (Distribution of Flowers)	858	651	643·5	± 8·555	+ 7·5
7. (Height of Plant) ...	1064	787	798	± 9·527	−11

Here are seven determinations of a frequency which is said to obey the law of Chance. Only one determination has a deviation from the hypothetical frequency greater than the probable error of the determination, and one has a deviation sensibly equal to the probable error; so that a discrepancy between the hypothesis and the observations which is equal to or greater than the probable error occurs twice out of seven times, and deviations much greater than the probable error do not occur at all. These results then accord so remarkably with Mendel's summary of them that if they were repeated a second time, under similar conditions and on a similar scale, the chance that the agreement between observation and hypothesis would be worse than that actually obtained is about 16 to 1.

The accuracy with which the theory fits the results obtained in the third hybrid generation may be tested in the same way. The plants of the second generation, which exhibit recessive characters, ought to produce offspring which

also exhibit only recessive characters, and Mendel assures us that this was the case; of those which exhibit dominant characters, one-third should produce apparently pure-bred offspring of dominant character, two-thirds should produce "hybrid" offspring. The behaviour of plants of the third generation, the offspring of plants with dominant characters, is shown in Table II.

TABLE II.

Proportion of Plants with Dominant Characters, among Hybrids of the Second Generation, which transmitted only Dominant Characters to their offspring.

Characters Crossed	Number of Second Generation Hybrids observed.	Number which transmitted only Dominant Characters	Theoretical number	Probable Error of Theoretical Expectation	Deviation of observed number
1. (Shape of Seeds) ...	565	193	188·3	±7·558	+4·7
2. (Colour of Cotyledons)...	519	166	173	±7·244	−7·0
3. (Colour of Seed-Coat) ...	100	36	33·3	±3·180	+2·7
4. (Shape of Pods)	100	29	33·3	±3·180	−4·3
5. (Colour of Pods)	100	40	33·3	±3·180	+6·7
6. (Distribution of Flowers)	100	33	33·3	±3·180	−0·3
7. (Height of Plant) ...	100	28	33·3	±3·180	−5·3

Three of these seven determinations give results which differ from that indicated by Mendel's hypothesis by less than their probable errors; the difference between the actual result and that postulated by the hypothesis is equal to the probable error in one case, and greater in three cases; but a discrepancy as great as twice the probable error only occurs once. Here again, therefore, Mendel's statement is admirably in accord with his experiment.

Mendel made several series of observations to test the validity of his statement in cases involving more than one pair of differential characters. The number of possible combinations quickly becomes too great to deal with experimentally, and the most complicated case recorded is that of hybrids between female parents of a race producing round smooth seeds with yellow cotyledons and grey-brown seed-coats, and male parents of a race with angular green seeds and white seed-coats. The original hybrids were 24 in number, and from these 639 hybrids of the second generation were grown and observed. Denoting the dominant characters, roundness, yellowness, and greyness of seed-coat, by A, B, C, and the corresponding recessive characters, angularity, greenness, and whiteness of seed-coat, by a, b, and c respectively, it is clear that the possible combinations of these characters in the hybrids of the second generation are the possible combinations of the three sets of characters

$$AA + aA + Aa + aa,$$
$$BB + bB + Bb + bb,$$
$$CC + Cc + cC + cc.$$

Since there are four possible combinations of each pair of characters, the number of possible combinations of the three pairs of characters is clearly $4^3 = 64$; and if we consider the union of male dominant with female recessive gametes to be the same as that of female dominant and male recessive, the number of different combinations is 27. Of these 27 different combinations, eight will each occur once, 12 will each occur twice, six will each occur four times, and one will occur eight times, in 64. The combinations actually deduced from the behaviour of the hybrids of the second generation and their offspring occurred with the following frequencies in the 639 plants:

TABLE III.

Frequency of the various possible combinations of Characters in Hybrids of the Second Generation from Races which differed in three Characters.

Combination	Frequency	Combination	Frequency	Combination	Frequency
AABBCC	8	*AABBCc*	22		
AABBcc	14	*AAbbCc*	17	*AABbCc*	45
AAbbCC	9	*aaBBCc*	25	*aaBbCc*	36
AAbbcc	11	*aabbCc*	20	*AaBBCc*	38
aaBBCC	8	*AABbCC*	15	*AabbCc*	40
aaBBcc	10	*AABbcc*	18	*AaBbCC*	49
aabbCC	10	*aaBbCC*	19	*AaBbcc*	48
aabbcc	7	*aaBbcc*	24		
		AaBBCC	14	*AaBbCc*	78
		AaBBcc	18		
		AabbCC	20		
		Aabbcc	16		

These numbers are all correlated, so that the system of results must be judged as a whole. Applying the method of Pearson (No. 25) the chance that a system will exhibit deviations as great as or greater than these from the result indicated by Mendel's hypothesis is about 0·95 (see Elderton, this Journal, *ante*, p. 161), or if the experiment were repeated a hundred times, we should expect to get a worse result about 95 times, or the odds against a result as good as this or better are 20 to 1.

Mendel's statements are based upon work extending over eight years. The remarkable results obtained are well worth even the great amount of labour they must have cost, and the question at once arises, how far the laws deduced from them are of general application. It is almost a matter of common knowledge that they do not hold for all characters, even in Peas, and Mendel does not suggest that they do. At the same time I see no escape from the conclusion that they do not hold universally for the characters of Peas which Mendel so carefully describes. In trying to summarise the evidence on which my opinion rests, I have no wish to belittle the importance of Mendel's achievement. I wish simply to call attention to a series of facts which seem to me to suggest fruitful lines of enquiry.

II.—*Other Evidence concerning Dominance in Peas.*

It is certain that an alternative inheritance, which may produce something like Mendel's phenomenon of segregation, occurs as a result of crossing races of animals and plants, when nothing comparable with dominance can be observed in the immediate offspring of the cross. The two phenomena must therefore be considered separately (cf. Correns, No. 5).

The evidence concerning dominance in the first hybrid generation of Peas relates chiefly to the colour of cotyledons and seed-coat, and to the shape of the hybrid seed. In judging it, we must be careful to realise what the statement, that a character is dominant, really means. Many races of Peas are exceedingly variable, both in colour and in shape. A race with "round smooth" seeds, for example, does not produce seeds which are exactly alike; on the contrary, many seeds of such a race as *Victoria*, used by Rimpau (No. 26) as a typically round and smooth-seeded Pea, or *Express* used by Tschermak (No. 27) in the same way, show very considerable irregularities; while in races such as *Prince of Wales* or *Telephone*, used by Tschermak and others as types of races with wrinkled seeds, hardly any two seeds are alike. So that both the category "round and smooth" and the category "wrinkled and irregular" include a considerable range of varieties. At the same time, the categories are undoubtedly often discontinuous, the most wrinkled seed of such a race as *Express* or *Victoria* being so much smoother and more rounded than the most regular seed of the typically "wrinkled" races, that no one who knows both races would hesitate for a moment in deciding which race a given seed resembled.

The statement that smoothness of seed is dominant over the production of wrinkles means therefore that if a parent, belonging to a variable race which falls into the category "smooth-seeded," be crossed with a parent belonging to a variable race of the category "with wrinkled seeds," the offspring will themselves be variable, but will always belong to the category "smooth-seeded": and as it is with shape, so it is with colour.

The first detailed account of the *colour* in hybrid Peas, which I have been able to find, is that of Goss, 1848 (No. 15), who fertilised the flower of a "blue-seeded" race with pollen of a race with yellowish white seeds. The hybrid seeds were all yellowish white; and the plants raised from them "produced some pods with all blue, some with all white, and many with blue and white seeds in the same pod"; or the result was probably that obtained by Mendel. In a note to this paper, the then Secretary of the Horticultural Society says that Mr Alexander Seton crossed the flowers of *Dwarf Imperial* "a well-known green variety of the Pea," with the pollen of "a white free-growing variety." Four hybrid seeds were obtained, "which did not differ in appearance from the others of the female parent." These seeds therefore did *not* obey the law of dominance, or if the statement be preferred, greenness became dominant in this case. The seeds were sown, and produced plants bearing "green" and "white" seeds side by

side in the same pod. An excellent coloured figure of one of these pods is given (*loc. cit.* Plate 9, Fig. 1), and is the only figure I have found which illustrates segregation of colours in hybrid Peas of the second generation. In 1849 Gärtner (No. 12, pp. 81—85) described the result of crossing races of Peas; the yellow-seeded *Pariser Wachserbse* which he calls *Pisum sativum luteum*, was fertilised (*a*) with pollen of *P. sativum macrospermum* which had seeds of an impure greenish-yellow, and (*b*) with pollen of the green-seeded *P. sativum viride*. In the first case the hybrid seeds were all pure yellow; in the second case twelve seeds were produced in four hybrid pods; and these were all of a greenish-yellow colour, although the greenish tinge disappeared from some of them on drying. Another yellow-seeded Pea (*P. sativum nanum repens*) fertilised with the pollen of the green-seeded *P. sativum viride* gave five hybrid pods with seeds, of which one contained five dirty green seeds, a second had five seeds which were "not distinctly yellow, but yellowish green," although they lost their greenness after being dried for two months. The others were not yellow like the mother, but "dirty yellow." It is clear that the "greenness" of *P. sativum viride* did in these cases affect the colour of the seeds, when its pollen was used to fertilise plants of yellow-seeded race, though it is difficult to judge exactly how great the effect was. The disappearance of the green colour on drying may have been simply due to the development of an air-space between the seed-coat and the cotyledons, which as Tschermak (No. 27) has pointed out may obscure the colour of the cotyledons in the dry seed, or it may have been due to a real increase in yellowness. The flowers of *P. sativum viride* were fertilised with pollen from two pure yellow-seeded races (*P. sativum luteum* and *P. sativum nanum repens*), and in the first case the seeds produced were "not distinctly yellow, still less blue or green, but dirty yellow." In the second case the seeds produced were yellow; and when *P. sativum viride* was fertilised by *P. sativum macrospermum* the seeds produced were apparently yellower than those of the male parent.

The effect of the dominance of yellowness over greenness of cotyledons in hybrids of the first generation, if it were indeed universal, would be so striking that one can hardly conceive that the great founders of our modern races of Peas, such as Knight (No. 18) and Laxton (No. 19), should have failed to notice it, and yet neither of them, in describing experience gained from crossing great numbers of plants, says that it is of general occurrence.

The colour of seeds of the first hybrid generation has not, so far as I can discover, been carefully described between the time of Mendel and the year 1893, when Giltay (No. 14), who does not appear to have known Mendel's work, crossed several yellow-seeded peas with the green-seeded race *Reading Giant*, and found that the colour of the cotyledons was always yellow, showing that Mendel's law of dominance was completely valid in this case. (Giltay points out the necessity of removing the seed-coats before determining the colour of the cotyledons; this is also emphasised by Tschermak, and I have found it quite necessary even in seeds of pure race.) In 1900 general attention was directed to

Mendel's work, and the result of crossing Peas has lately been described by Correns (Nos. 3 and 4) and in greater detail by Tschermak (Nos. 27 and 28). Correns confirms Mendel's statements concerning the dominance of yellow cotyledons, but Tschermak makes a more detailed statement, which does not so fully agree with Mendel. It is not quite easy to follow Tschermak's account, because he does not describe all his very numerous and careful experiments in such a way that one can be sure how many hybrid peas he observed. He certainly crossed between 80 and 100 flowers, belonging to green or yellow-seeded races, with pollen from plants with seed of the opposite colour, obtaining between 300 and 400 hybrid seeds. Of these hybrid seeds about 40 were not distinctly yellow, so that 90 per cent. of the hybrids exhibited the dominant character. Of the seeds which did not exhibit this character, some were yellow with green patches, eight were green, four were "yellowish green," and five are described as showing "Grün und Uebergänge von Gelb zu Grün," so that while some 10 per cent. of the hybrids did not exhibit dominance of yellow, some 2 or 3 per cent. exhibited a close approximation to the character of the "recessive" parent. A further case is recorded by Tschermak, where a plant of the yellow-seeded variety *Buchsbaum*, growing in the open, produced a pod in which every pea except one was green, the exception showing a little yellow. One of the green seeds was sown, and the plant produced fifteen yellow and three green seeds, thus behaving like a hybrid. Tschermak considers that this is a case of accidental crossing, with dominance of green, and although some of the green peas, produced in his experiments, may have been due to accidental self-fertilisation, he regards some of them, at least, as hybrids (cf. No. 28, pp. 663—664). It should be said that some of the crosses referred to were made between the (yellow-seeded) *P. arvense*, var. *Graue Riesen*, and green-seeded varieties of *P. sativum*. In all such crosses the law of dominance of yellow held absolutely.

These results show clearly enough that the law of dominance is, as Tschermak says, not absolutely true of cotyledon colour, and as will presently be shown, the exceptions to the law, which he observed, form a very large percentage of the total result obtained when certain races were crossed.

The case of cotyledon colour has been considered first, because the evidence with regard to it is more favourable to Mendel than is the evidence touching other seed characters. The *shape* of seeds, whether smooth and rounded or irregular and wrinkled, is even more difficult to express in words than the colour of their cotyledons, and the varieties appear to be even less constant. The evidence against the universal validity of the law of dominance is here much stronger than in the case of cotyledon-colour. Two striking cases were observed by Rimpau (No. 26, pp. 36, 37), who crossed the smooth-seeded race *Victoria* with two wrinkled races, *Knight's Marrow* and *Telephone*. He made each cross in both the possible ways, and found the second hybrid generation dimorphic, as usual. From the cross *Victoria* ♀ × *Knight's Marrow* ♂ he obtained round and wrinkled seeds of the second generation, as Mendel's statements would lead us to expect. The wrinkled

seeds were sown, and produced both round and wrinkled seeds, the wrinkled (recessive) character becoming "true" only in the fifth year. The wrinkled seeds of the second generation from the cross *Victoria* ♀ × *Telephone* ♂ also gave rise to plants with rounded and wrinkled seeds; but the descendants of these were not observed. If Mendel's law of segregation was here followed, his law of dominance was not; because if both are valid, a plant of the second hybrid generation which exhibits recessive characters must behave, so far as those characters are concerned, like a pure-bred individual. The most careful account of seeds of the first hybrid generation is that of Tschermak, in the papers already quoted. The variety *Telephone* especially, when crossed with smooth-seeded varieties, gave a large number of wrinkled seeds, or seeds of intermediate character. Thus the *Pois d'Auvergne* used by Tschermak had the ripe seeds "always round and smooth"; but out of 27 seeds recorded from seven crosses between this pea as the ♀ and *Telephone* as ♂ parent, nine were slightly wrinkled, and two out of ten resulting from the reciprocal cross were slightly wrinkled. These are difficult cases to judge; but the cross *Telephone* ♀ × the smooth-seeded *Plein le Panier* ♂ gave seeds which "differed only slightly from those of the mother," although the wrinkles were perhaps not so deep. Again, *Telephone* ♀ × the smooth *grünbleibende Folger* ♂ gave seeds which certainly belonged to the category "wrinkled," although they were more rounded than those of pure *Telephone*. After pointing out the exceptions to dominance of yellow cotyledon-colour already mentioned, Tschermak summarises his view of the dominance of wrinkles as follows: "Auch in Bezug auf die Form fehlt es nicht an Fällen, in denen sich das "sonst dominirende mit dem sonst recessiven Merkmale in einem gewissen "Verhältnisse combinirte. So ergiebt *im Allgemeinen* die Bestäubung einer "gerunzelten Markerbse mit Pollen einer glattsämigen Varietät abgerundete, "schwach gerunzelte Produkte, und zwar eher als die umgekehrte Verbindung "der Eltern."

This is a sufficient statement of Tschermak's result, so far as it concerns races of *P. sativum*. Correns (No. 3) found that the *grüne späte Erfurter Folger-Erbse* which has round smooth seeds, was not dominant in this respect over the *Pahl-Erbse mit purpurrothen Hülsen*.

The foregoing crosses show that the law of dominance does not always hold for the shape of seeds, when the races crossed belong to the species *P. sativum*. The race of *P. arvense* called *Graue Riesen* is held by some botanists to belong to a different "species," and may therefore be considered separately. Tschermak has crossed *Graue Riesen* with five races of *P. sativum*, and he finds that the form of the first hybrid seeds *follows the female parent*, so that if races of *P. sativum* with round smooth seeds be crossed with *Graue Riesen* (which has flattened, feebly wrinkled seeds), the hybrids will be round and smooth or flattened and wrinkled, as the *P. sativum* or the *Graue Riesen* is used as female parent. There is here a more complex phenomenon than at first sight appears; because if the flowers of the first hybrid generation are self-fertilised, the resulting seeds of the

second generation invariably resemble those of *P. arvense* in shape, although in colour they follow Mendel's law of segregation!

The discussion of *seed-coat colour* is more difficult than that of the two characters already dealt with, because the recent writers (especially Tschermak) pay attention to a character, namely green pigment in the seed-coat, which Mendel does not mention. It is clearly unreasonable to expect that every character in the seed-coat should obey the same law, and therefore in trying to estimate the agreement between Mendel and later workers, many valuable data collected by Tschermak will be neglected. The most striking exception to the law of dominance is that observed by Correns. The *grüne späte Erfurter Folger-Erbse* has a nearly colourless seed-coat (recessive, Mendel); the *Purpurviolett-schottige Kneifel-Erbse* and the *Pahl-Erbse mit purpurrothen Hülsen* have the seed-coat uniformly orange, becoming brown with age. In hybrids of the first generation, between either of these last varieties and the *Erfurter Folger-Erbse*, the coats of seeds (often in the same pod) were sometimes nearly colourless, sometimes intensely orange-red, but generally more or less orange-red, and spotted more or less strongly with blackish violet. The seeds of extreme colour, those with orange or those with almost colourless seed-coats, gave plants of the second generation which again showed the same extremes of colour in the seed-coats, connected by transitional forms. This is clearly a case in which one of Mendel's characters obeys neither the law of dominance nor the law of segregation.

The appearance of blackish-violet dots on the seed-coats of similar hybrids was noticed by Mendel. In his crosses however one parent showed traces of such dots, which were only exaggerated in the offspring. In the plants used by Correns as parents, the seeds with brown coats are not said to have possessed these dots. An exaggeration of the purple dots, of a kind similar to that observed by Mendel, but even greater in degree, was found by Tschermak in crosses between *Graue Riesen* and varieties of *P. sativum* with colourless seed-coats.

Enough has been said to show the grave discrepancy between the evidence afforded by Mendel's own experiments, and that obtained by other observers, equally competent and trustworthy. It does not seem to me reasonable to doubt the substantial accuracy of any of the statements made by the observers quoted. I have deliberately refrained from collecting the numerous scattered records of odd crosses which exist in the various journals devoted to horticulture, because many of these are published either anonymously or by persons whose skill in performing the difficult operation of crossing in such a way as to be sure of the result may possibly be doubted. The evidence brought together rests upon the statements of men whose knowledge and skill are beyond question, and the only conclusion which can, I think, be fairly drawn from it is, that dominance of any of the characters mentioned is not an invariable attribute of the character, but that a cross between pairs of parents, such that the different members of each pair differ to the same extent in cotyledon colour or in similar characters, may in different

cases lead to widely different results, and one great reason why this is so becomes evident on a little consideration.

Mendel treats such characters as yellowness of cotyledons and the like as if the condition of the character in two given parents determined its condition in all their subsequent offspring. Now it is well known to all breeders, and is clearly shown in a number of cases by Galton and Pearson, that the condition of an animal does not as a rule depend upon the condition of any one pair of ancestors alone, but in varying degrees upon the condition of all its ancestors in every past generation, the condition in each of the half-dozen nearest generations having a quite sensible effect. Mendel does not take the effect of difference of ancestry into account, but considers that any yellow-seeded Pea, crossed with any green-seeded Pea, will behave in a certain definite way, whatever the ancestry of the green and yellow peas may have been. [He does not say this in words, but his attempt to treat his results as generally true of the characters observed is unintelligible unless this hypothesis be assumed.] The experiments afford no evidence which can be held to justify this hypothesis. His observations on cotyledon colour, for example, are based upon 58 cross-fertilised flowers, all of which were borne upon ten plants; and we are not even told whether these ten plants included individuals from more than two races. The many thousands of individuals raised from these ten plants afford an admirable illustration of the effect produced by crossing a few pairs of plants of known ancestry; but while they show this perhaps better than any similar experiment, they do not afford the data necessary for a statement as to the behaviour of yellow-seeded peas in general, whatever their ancestry, when crossed with green-seeded peas of any ancestry.

When this is remembered, the importance of the exceptions to dominance of yellow cotyledon-colour, or of smooth and rounded shape of seeds, observed by Tschermak, is much increased; because although they form a small percentage of his whole result, they form a very large percentage of the results obtained with peas of certain races. The fact that *Telephone* behaved in crossing on the whole like a green-seeded race of exceptional dominance shows that something other than the mere character of the parental generation operated in this case. Thus in eight out of 27 seeds from the yellow *Pois d'Auvergne* ♀ × *Telephone* ♂ the cotyledons were yellow with green patches; the reciprocal cross gave two green and one yellow-and-green seed out of the whole ten obtained; and the cross *Telephone* ♀ × (yellow-seeded) *Buchsbaum* ♂ gave on one occasion two green and four yellow seeds.

So the cross *Couturier* (orange-yellow) ♀ × the green-seeded *Express* ♂ gave a number of seeds intermediate in colour. [It is not quite clear from Tschermak's paper whether *all* the seeds were of this colour, but certainly some of them were.] The green *Plein le Panier* ♀ × *Couturier* ♂ in three crosses always gave either seeds of colour intermediate between green and yellow, or some yellow and some green seeds in the same pod. The cross reciprocal to this was not made; but *Express* ♀ × *Couturier* ♂ gave 22 seeds, of which four were yellowish-green.

These facts show *first* that Mendel's law of dominance conspicuously fails for crosses between certain races, while it appears to hold for others; and *secondly* that the intensity of a character in one generation of a race is no trustworthy measure of its dominance in hybrids. The obvious suggestion is that the behaviour of an individual when crossed depends largely upon the characters of its ancestors. When it is remembered that Peas are normally self-fertilised, and that more than one named variety may be selected out of the seeds of a single hybrid pod, it is seen to be probable that Mendel worked in every case with a very definite combination of ancestral characters, and had no proper basis for generalisation about yellow and green peas of any ancestry.

Now in such a case of alternative inheritance as that of human eye-colour, it has been shown that a number of pairs of parents, one of whom has dark and the other blue eyes, will produce offspring of which nearly one half are dark-eyed, nearly one half are blue-eyed, a small but sensible percentage being children with mosaic eyes, the iris being a patchwork of lighter and darker portions. But the dark-eyed and light-eyed children are not equally distributed among all families; and it would almost certainly be possible, by selecting cases of marriage between men and women of appropriate ancestry, to demonstrate for their families a law of dominance of dark over light eye-colour, or of light over dark. Such a law might be as valid for the families of selected ancestry as Mendel's laws are for his peas and for other peas of probably similar ancestral history, but it would fail when applied to dark and light-eyed parents in general,—that is, to parents of any ancestry who happen to possess eyes of given colour.

This neglect of ancestry, the tendency to regard offspring as resembling their parents rather than their race, accounts for much of the apparent inconsistency between the results obtained by different observers who have crossed plants or animals.

The writer who has most clearly recognised the importance of ancestry in connection with Mendel's work is Correns. In a recent summary of his views (No. 6) and in his fuller account of experiments in crossing races of Maize (No. 7) he says that between the complete equivalence of two characters and the complete dominance of one of them, all intermediate stages may exist, and that the dominance of a character varies (*a*) according to the individuality of different gametes from the same gonad; (*b*) according to the individuality of the different plants of the same race; (*c*) according to the race of the plant. He also points out that the offspring of pure-bred races differ in their power of transmitting "alternative" characters (which Correns, adopting Mendel's theory of the constitution of gametes in this case, calls schizogonic) from the offspring of a cross, although the apparent characters of both may be the same. Correns illustrates and justifies his statements by detailed accounts of experiments with Maize; and his figures of particoloured heads of seed, produced by fertilising a white-seeded female flower with pollen from a single male flower of blue-seeded race are most striking. They show that the result may be either a blue seed, or a white seed, or

a piebald white and blue seed, or a seed of uniform but intermediate colour. The peculiar process of double fertilisation to which this change of colour in the extra-embryonic tissues of Maize is due does not seem to me to affect the relevance of the observations.

In order to emphasize the need that the ancestry of the parents, used in crossing, should be considered in discussing the results of a cross, it may be well to give one or two more examples of fundamental inconsistency between different competent observers.

Correns (No. 5) himself has crossed the Stock *Matthiola annua* = *M. incana* D.C., which has its green parts covered with grey hairs, and the smooth-leaved Stock, *M. glabra* D.C. The 111 hybrids obtained from reciprocal crosses all had hairy leaves. Trevor Clarke made a similar cross, using of course different races, and found that half his seedlings had smooth leaves, half had hairy leaves (No. 2).* Again, the petals of the *M. incana* used by Correns were violet, those of the *M. glabra* were yellowish white. The petals of the hybrid seedlings were invariably either violet of the shade of *M. incana*, or violet more or less obviously spotted with pale violet. On the other hand, Nobbe (No. 23) crossed a number of varieties of *M. annua* in which the flowers were white, violet, carmine-coloured, crimson, or dark blue. These were crossed in various ways, and before a cross was made the colour of each parent was matched by a mixture of dry powdered colours, which was preserved. In every case the hybrid flower was of an intermediate colour, which could be matched by mixing the powders which recorded the parental colours. The proportions in which the powders were mixed are not given in each case, but it is clear that the colours blended.

Again, de Vries (Nos. 30 and 31) whose evidence for most of his statements concerning the validity of the law of dominance has not yet been published, crossed *Datura tatula* with the smooth-fruited form of *D. stramonium*. *D. tatula* has blue or violet flowers, and dull purple stems; the flowers of *D. stramonium* are white and its stems bright green. De Vries found that in all his hybrid seedlings the flowers were blue, and he concludes that the colouring of *D. tatula* is dominant in Mendel's sense. Naudin (No. 22) crossed *D. tatula* with the variety of *D. stramonium* which has spiny fruit and white flowers. The flowers of his hybrids were violet, but paler than those of *D. tatula*, and their stems were flushed with purple, but to a less extent than those of *D. tatula*; so that in his crosses a blending of the parental colours occurred. Again, de Vries found that the fruit of his hybrids was spiny, and concludes that the production of spines is a dominant character. Naudin crossed the smooth-fruited and the spiny-fruited forms of *D. stramonium* (which he called distinct "species") and found that among the fruits of forty hybrids some were completely spiny; others were smaller than the normal fruits of the spiny form (in this resembling the smooth form), while their spines were shorter and weaker than in the pure spiny form; others again were a

* According to Bateson (*Journ. Roy. Hort. Soc.* Vol. XXIV. p. 64), Miss Saunders has obtained a similar result. Correns doubts whether Trevor Clarke's flowers were completely cross-fertilised.

mosaic of smooth and spiny segments. On the other hand, *D. stramonium* of the spiny form, crossed with *D. ceratocaula*, which has a smooth fruit of different character, gave hybrids whose fruits exactly resembled those of *D. stramonium*.

Examples might easily be multiplied, but as before, I have chosen rather to cite a few cases which rest on excellent authority, than to quote examples which may be doubted. I would only add one case among animals, in which the evidence concerning the inheritance of colour is affected by the ancestry of the varieties used. Many people have crossed the various white, piebald and wild-coloured varieties of the rat (*Mus decumanus*), the closely similar varieties of the mouse (*Mus musculus*) have also been frequently crossed. In both rats and mice von Fischer (Nos. 10 and 11) says that piebald rats crossed with albino varieties of their species, give piebald young if the father only is piebald, white young if the mother only is piebald. Crampe (No. 8) finds that in either case the offspring are a mixture of piebald forms and albinos. Results such as those which Crampe records in rats are commonly obtained when piebald and albino mice are paired; but both Haacke (No. 17) and von Guaita (No. 16) find that when the ordinary European albino mouse is paired with the piebald Japanese "dancing" mouse, the offspring are either like wild mice in colour, or almost completely black. Again, Crampe says that when white and wild-coloured mice are crossed the offspring are invariably coloured like wild mice. Colladon is reported by Prévost and Dumas (*Ann. Sci. Nat.* I., 1824) to have obtained both albino and wild-coloured individuals from similar crosses, but no piebald individuals.

These examples, chosen from many others which might have been cited, seem to me to show that it is not possible to regard dominance as a property of any character, from a simple knowledge of its presence in one of two individual parents. The degree to which a parental character affects offspring depends not only upon its development in the individual parent, but on its degree of development in the ancestors of that parent. A collection of cases which illustrate this point is given by Bateson (No. 1).

III.—*The Hybrid Peas of the Telephone Group.*

If Mendel's statements were universally valid, even among Peas, the characters of the seeds in the numerous hybrid races now existing should fall into one or other of a few definite categories, which should not be connected by intermediate forms. In attempting to follow the results obtained by Tschermak and others I have carefully examined the seed characters of some twenty named varieties, and the present condition of many I have studied seems to me quite incompatible with the general validity of Mendel's statements. The aberrant behaviour of the race called *Telephone* has already been observed by Rimpau and Tschermak, and I have therefore endeavoured to learn the past history and the present condition of this Pea and the races allied to it. In my attempt I have received very great help from Messrs Carter and Co., who originated *Telephone* and its allies, from Messrs Sutton and Sons, and from Messrs Vilmorin-Andrieux and Co.,

of Paris. All these gentlemen have supplied me with quantities of seeds of various races, and have spent much time and trouble in answering questions concerning them. I gladly take this opportunity of expressing my gratitude to all of them, and especially to Messrs Carter and Co., the value of whose help will be presently apparent.

In 1876 Culverwell introduced into the English market a Pea which he called *Telegraph*. This was a hybrid race, and he says that his stock was derived from a single cross-fertilised flower. The stock of *Telegraph* was ultimately bought by Carter and Co., who found it so variable that without further crossing they produced from it, by simple selection, the four races now known as *Telephone*, *Stratagem*, *Pride of the Market*, and *Duke of Albany*. These four races, together with *Telegraph*, are still cultivated, and I have examined them all. Their history is well known, and is authenticated not only by Carter's records, but by letters of Culverwell, and by Laxton (No. 20). Between 1880 and 1890 Carter and Co. crossed *Stratagem* with *Giant Marrow*, another of Culverwell's races; and the offspring of this cross was introduced into the market in 1892 as *Daisy*. A few years later the race *Daisy* was crossed with *Lightning*, a Pea which has round, nearly smooth seeds, and yellow cotyledons. The result of this cross is known as *Early Morn*. Culverwell's *Giant Marrow* is not now easy to obtain; but all the other races referred to are grown commercially, on a large scale, and I have examined the characters of their seeds.

In attempting to judge the results of other observers, including those of Mendel himself, I have constantly found it difficult to understand the statements made, because of the vagueness of the terms used to describe shape and colour. In order to make my own statements about colour as intelligible as may be, I selected from a sample of *Telephone* grown by Carter a series of 18 peas, which show, after removal of the seed-coats, a fairly gradual series of transitional colours from a deep green to an orange yellow. These seeds were arranged in groups of three, so that six colour-groups were formed; and these groups, numbered from one to six, the first being green and the sixth orange-yellow, form a scale in terms of which I shall express the colour of all the peas described. The groups are represented in Figs. 1—6 on Plate I. [Peas of all these colours are numerous, as are peas exhibiting the other characters shown on the plate: and I shall have much pleasure in sending a replica of the set from which the plate was photographed to any naturalist who may write to me before my stock is exhausted.]

In the same way, I have tried to make my statements about the shape of seeds intelligible by means of the photographs reproduced on Plate II.*

(1) *Telegraph* was at first very variable, both in shape and in colour, and *Telephone* did not become thoroughly differentiated from it for some years. In 1878 the Royal Horticultural Society gave a certificate to *Telephone* as a

* Plates I. and II. will be found facing p. 254.

new variety, and in 1879 a controversy arose as to the reality of the distinction between the two. The Editor of the *Gardener's Chronicle* (1879, p. 210) says, "It is clear that what Messrs Carter and Co. are sending out as *Telephone* is "the most wrinkled seed selected from *Telegraph*," and later in the same year (August 2nd, p. 146) the Editor describes seeds sown at his direction in order to test the identity of the two races. Among the seeds sown are three samples of *Telegraph*. In one the seeds are said to be "mixed, round and wrinkled," in the second and third they are "but slightly wrinkled." And since on the one hand the seeds of *Telephone* are admittedly "wrinkled," while on the other hand there is some doubt whether *Telegraph* and *Telephone* are really distinct, it is clear that the offspring of the hybrid *Telegraph* had not become sharply divided into a smooth-seeded and a wrinkled race in 1879, as they should have done on Mendel's hypothesis. The presence of forms intermediate between the categories "round" and "wrinkled" in 1879 seems proved by the fact that the controversy referred to arose. The *Telegraph* of to-day is more than 25 generations removed from the original cross, and it should, on Mendel's view, have split into two groups, one with seeds completely smooth and rounded, the other with wrinkled seeds. Seeds of intermediate type should not occur, and only one plant out of every $2^{25}-1$, or say one plant in every thirty-three millions, should bear seeds of both kinds.

As a matter of fact, the seeds of *Telegraph* as grown by Carter and Co. exhibit every conceivable condition between the perfectly smooth forms shown in Fig. 1, Plate II., and the forms with well-marked wrinkles shown in Fig. 4 on the same plate. The groups photographed in Figs. 1—4 were chosen from a single quart of *Telegraph* given to me by Messrs Carter; and a rough estimate of the relative frequency with which the characters of each group occurred in a small sample gave the following result:

Group 1.....	242
Group 2.....	228
Group 3.....	28
Group 4.....	2
	500

It is not suggested that the groups differ successively by approximately equal increments of wrinkles, so that the scale they form is certainly imperfect. The grouping of the peas is also very rough. But with all these defects the study of *Telegraph* seems to me to demonstrate that the offspring of a hybrid Pea, 25 generations at least after crossing, may contain a large percentage of individuals which on Mendel's view that roundness is dominant ought never to occur at all.

The distinction in colour between *Telegraph* and *Telephone* was also discussed twenty years ago. The seed-coats are often so opaque in all these races, that it is difficult to estimate cotyledon colour from descriptions of the pea

with its seed-coat uninjured. A statement by Culverwell seems however worth quoting. In 1882 he writes of *Telegraph* and *Telephone*: "The two will always "come from one sort, more especially from the green variety." And a line or two later, describing the result of sowing what he thought a good sample of *Telegraph*, he says, "Strange to say, although the peas were taken from one lot, those sown "in January produced a great proportion of the light variety known as *Telephone*. "These were of every shade of light green up to white, and could have been "shown for either variety." (*Gardener's Chronicle*, July 1882, p. 150.)

This is only one of a number of statements, scattered through the *Gardener's Chronicle* from 1879 onwards, which show that neither *Telegraph* nor *Telephone* was recognised as sharply divided into green or blue and yellow or white races during the early years of their existence; Culverwell's statement that the green variety was especially variable in colour in 1882, shows more than this, however, because it shows that in his opinion a plant of a late generation, which exhibited a "recessive" colour, did not produce exclusively recessive offspring, as Mendel says it should. At the present day they are recognisably distinct, both in shape and in cotyledon colour; but the colour of the cotyledons is variable in both. Using the groups of seeds photographed on Plate I. as a scale, I found among 576 seeds of *Telegraph*, 512 seeds of fairly uniform colour, and 64 obviously piebald seeds. The self-coloured seeds fell into one or other of the colour groups from 1 to 6, with frequencies given in Table IV. (p. 250). The piebald remainder were generally half bright yellow and half bright green. There were generally only two patches of colour on each seed, and one cotyledon was often as yellow as Group 6, Plate I., while the other was as green as Group 1.

(2) *Pride of the Market.* If shape and colour be considered together, this is one of the most constant races selected from the offspring of *Telegraph*. Of 500 seeds whose shape was examined, there were

Like Fig. 1.....	260
„ Fig. 2.....	218
„ Fig. 3.....	20 or 21
„ Fig. 4.....	2 or 1

while the colour, as shown in Table IV., is distinctly less variable than in *Telegraph*, and the percentage of piebalds perhaps significantly smaller.

(3) *Stratagem* is nearly as constant as *Pride of the Market* in colour, but it is more variable in shape. The seeds are larger than those of any other race belonging to this group, and are normally much flattened; for this reason it is difficult to compare the seeds of *Stratagem* with those of the races just described. Three groups of *Stratagem* seeds are photographed in Figs. 7, 8 and 9 (Plate II.) and they show the characteristic shapes and wrinkles fairly well. An inspection of Figs. 1—9 will show better than any words, *first* how completely the categories "smooth" and "wrinkled" pass into each other in these races;

and *secondly* how enormously the range of variation included under the two differs.

The colour of *Stratagem* is interesting because of its intermediate condition between the extremes observed in the group of races to which it belongs. As will be seen from Table IV., piebald seeds are rarer than in any of its allies, and the great majority of seeds fall into the colour-groups 2 and 3*, with a very small percentage of seeds so green as Group 1, or as yellow as Groups 4 and 5. The general impression produced by a mass of these seeds, when stripped of their coats, is certainly green; but it is a far yellower green than that of either *Telegraph* or *Pride of the Market*; and I think there can be no question that in *Stratagem* a blend of green and yellow has been inherited, and fixed by a process of selection.

(4) *Telephone*, in the samples given to me by Carter and Co. or by Sutton and Sons, is fairly uniform in shape, being practically always as wrinkled as Figs. 5 and 6. It appears however from Tschermak's account (No. 27, p. 480) that samples which he bought in Ghent and in Quedlinburg were more variable, and contained smoother, more oblong forms. The same thing is true of a sample which I bought in Oxford from Messrs McGreal and Co. The colour of all the samples I have seen (from Vilmorin-Andrieux, Carter, Sutton, and McGreal) is exceedingly variable. Every one of the six colour-groups is abundantly represented, and indeed the scale of colours, which was found suitable for the whole series of races, was selected from this race alone. There can be no question that self-coloured seeds of every shade intermediate between the darkest green of Fig. 1 and the most intense orange-yellow of Fig. 6 occur in every large sample of this race, while piebald peas, with every degree of difference between the colours of their patches, occur also. Groups of piebalds, illustrating their main characters, are shown in Figs. 13—18 on Plate I. The colour variation of this race, together with Tschermak's description of its behaviour when crossed as a green-seeded pea with exceptional dominance, led me to tabulate the colours in several samples. The results are given in Table IV.; and while the different samples differ considerably among themselves, they all show the same general characters. In all of them the range of colour is the whole range of the scale, and the groups of intermediate colour occur most frequently; in all of them there is a large percentage of piebald individuals. It is evident that in a very large percentage of these individuals the characters "green" and "yellow" are perfectly blended, while others present a mosaic or piebald character; but even in these piebald individuals the patches of colour cannot, in some cases, be spoken of as pure green or pure yellow; the pea may exhibit a blend of green and yellow on every part of its surface, but the proportions in which the two are mixed may differ in the two cotyledons, or in different parts of the same cotyledon. The suggestion that yellow and green are mutually exclusive alternatives, or that

* As will be seen from Plate I., Figs. 7—12, many seeds of this race are intermediate in colour between Figs. 2 and 3.

anything like Mendel's phenomena of dominance and segregation has occurred in the history of *Telephone*, is clearly absurd.

(5) *Duke of Albany* resembles *Telephone* so closely in the general characters of its seeds, that I have only examined a small sample, sent to me by Messrs Sutton and Sons. The frequency of colour varieties in this sample is given in Table IV.

(6) *The Hybrid Descendants of Stratagem.* It has already been pointed out that the average colour of *Stratagem* must be regarded as a blend of yellow and green. The yellow properties of this blend are well seen in *Daisy*, the result of the cross *Stratagem* × *Giant Marrow*. In shape, the seeds of *Daisy* are generally oval, with two opposite concave faces, the whole surface being wrinkled to a variable extent. Forms intermediate between this and the thinner flattened form of *Stratagem* also occur, as do more rounded forms. The chief shapes may be gathered from Figs. 9 and 10 (Plate II.). The average colour is a fairly uniform green; but as will be seen from Table IV., piebald individuals, and individuals of colour 4, occur.

Daisy and *Lightning* were crossed, as has been said, to produce the race *Early Morn*. In its present condition *Lightning* is a very round and smooth-seeded Pea, the shapes which occur being shown in Figs. 13 to 15 (Plate II.). The wrinkles of the group photographed in Fig. 15 occurred in about 16 out of 250 seeds examined. The colour varieties include a small percentage of green and piebald seeds; but the great majority of the seeds are of a deeper orange than that of Group 6. *Lightning* is what I imagine most of the writers on Mendel's laws would accept as a typical round, smooth, yellow Pea. The seeds of the second generation (produced by the first hybrid plants) were variable both in shape and colour. Messrs Carter and Co. have circulated reproductions of a photograph of one pod, containing peas of the second generation. The pod contains seven seeds, of which three are labelled "white," three "blue," and one "green." Two of the seeds seem smooth, while the rest are wrinkled in various degrees. By the kindness of Messrs Carter and Co. I have been able to examine a sample of *Early Morn* not as it is now, but as it was when they introduced it. The race was apparently selected from seeds of the second hybrid generation which were "recessive" both in colour and in shape; and although we must remember that the seed saved when making the stock from which a new race might be established was selected, and cannot be supposed to represent all the seed produced by the offspring of all the green and wrinkled hybrids, yet so far as the evidence available goes it favours the view that the offspring of wrinkled seeds were themselves wrinkled, for the sample given to me contains only wrinkled seeds of fairly uniform character. These seeds do not in the least resemble the seeds of *Daisy*. As will be seen from Figs. 16—18 (Plate II.), they are far more like those of *Telephone* in the character of their wrinkles than they are like either of their parents. This is another example of the danger incurred by

using Mendel's categories without careful examination. The statement that the seeds of *Daisy* and *Early Morn* are wrinkled and irregular, while those of *Lightning* are fairly smooth and rounded, is perfectly true; but if made in this form, without further explanation, it suggests a likeness between *Daisy* and *Early Morn* which does not exist. So far as the shape of *Early Morn* exhibits a reversion to any of its known ancestors, the reversion is directly to the most wrinkled type of the original *Telegraph*. In sending me the sample I have examined, Messrs Carter wrote of *Early Morn*: "You will clearly see the trace of "the white Pea which was one of its parents." The analysis of the colour-variations, given in Table IV., abundantly confirms Messrs Carter's statement: the majority of seeds are green; but seeds of intermediate colour, and piebald seeds, do occur with quite sensible frequency.

TABLE IV.

Frequency of Colour Variation.

Race	Colour 1	Colour 2	Colour 3	Colour 4	Colour 5	Colour 6	Piebald	Total
Telegraph	354	95	47	10	4	2	64	576
Pride of the Market	447	76	19	2	2	1	55	602
Stratagem	200	367	154	16	5	0	40	602
Telephone (Carter)...	191	289	195	59	38	38	133	943
„ (Sutton)...	13	83	112	32	15	13	43	311
„ (Vilmorin)	29	69	69	23	3	2	5	200
Duke of Albany ...	26	70	121	53	11	20	27	328
Daisy...	78	175	27	7	0	0	17	304
Early Morn	267	239	81	2	1	1	9	600

IV.—*Mendel's Law of Segregation.*

The history of the Telephone group of Peas is clearly inconsistent with the universal validity of Mendel's laws of dominance and segregation. The hybrid *Telegraph* produced seeds of various colours at the time of its origin, and now, more than five-and-twenty years after its introduction, it does so still. From the variable offspring of this hybrid races have been produced by selection, which bear either yellow-green seeds of fairly constant character (*Stratagem*) or a mixture of self-coloured and conspicuously piebald seeds, the self-coloured seeds presenting every conceivable colour between deep green and intense orange-yellow (*Telephone*). These races are many generations removed from their common hybrid ancestor, and it may be suggested that phenomena such as those described by Mendel could have been observed in the earlier generations. No decisive answer to such a suggestion can now be obtained; but the hybrids *Daisy* and *Early Morn* are both little removed from their cross-fertilised ancestor; in their present selected form both exhibit characters which should, on Mendel's view, be recessive, and should therefore produce invariably recessive offspring, while in fact their offspring is variable.

The behaviour of these races is much more exactly in accord with the statements of Laxton. In 1866, the year in which Mendel's paper was published in printed form, Laxton published a short summary of the results gained during many years of work upon Peas, which shows that his experience was altogether different from that of Mendel. He says of the colour and character of the seed:

"The results of experiments in crossing the Pea tend to show that the colour "of the immediate offspring or second generation sometimes follows that of the "female parent, is sometimes intermediate between that and the male parent, and "is sometimes distinct from both; and although at times it partakes of the colour "of the male, it has not been ascertained by the experimenter ever to follow the "exact colour of the male parent. In shape, the seed frequently has an inter-"mediate character, but as often follows that of either parent. In the second "generation, in a single pod, the result of a cross of Peas different in shape and "colour, the seeds are sometimes all intermediate, sometimes represent either or "both parents in shape or colour, and sometimes both colours and characters, with "their intermediates, appear. The results also seem to show that the third "generation, that is to say, seed produced from the second generation or the "immediate offspring of a cross, frequently varies from its parents in a limited "manner, usually in one direction only, but that the fourth generation produces "numerous and wider variations, the seed often reverting partly to the colour and "character of its ancestors of the first generation, partly partaking of the various "intermediate colours and characters, and partly sporting quite away from any of "its ancestry." (Laxton, No. 19, p. 156.)

In a review of his own work, nearly a quarter of a century later, Laxton says again: "By means of cross-fertilization alone, and unless it be followed by careful "and continuous selection, the labours of the cross-breeder, instead of benefiting "the gardener, may lead to utter confusion." (No. 20.)

These statements show that the phenomena of inheritance in cross-bred Peas, as Laxton observed them, were far more complex than those described by Mendel; but they do not preclude the possibility of a simple segregation, such as Mendel describes, in particular cases. Such cases of simple segregation have been described in Peas by both Correns and Tschermak in the papers already cited. The proportions of dominant and recessive individuals, found by Correns in his later generations, were in accord with Mendel's results; those found by Tschermak, as he himself points out, were extremely improbable, on Mendel's hypothesis.

Taking these results together with Laxton's statements, and with the evidence afforded by the Telephone group of hybrids, I think we can only conclude that segregation of seed-characters is not of universal occurrence among cross-bred Peas, and that when it does occur, it may or may not follow Mendel's law. The law of segregation, like the law of dominance, appears therefore to hold only for races of particular ancestry. In special cases, other formulae expressing segregation have

been offered, especially by De Vries (32) and by Tschermak (29) for other plants, but these seem as little likely to prove generally valid as Mendel's formula itself.

The fundamental mistake which vitiates all work based upon Mendel's method is the neglect of ancestry, and the attempt to regard the whole effect upon offspring, produced by a particular parent, as due to the existence in the parent of particular structural characters; while the contradictory results obtained by those who have observed the offspring of parents apparently identical in certain characters show clearly enough that not only the parents themselves, but their race, that is their ancestry, must be taken into account before the result of pairing them can be predicted.

ALPHABETICAL LIST OF AUTHORS CITED.

1. Bateson, W. On Progress in the Study of Variation. Science Progress, N. S., Vol. I. No. 5 (1897) and Vol. II. No. 6 (1898).
2. Clarke, Trevor. Report of Proceedings: International Horticultural Exhibition and Botanical Congress, London, 1866, p. 143.
3. Correns, C. G. Mendel's Regeln über das Verhalten der Nachkommenschaft der Rassenbastarde. Berichte d. deutsch. botan. Gesellsch. XVIII (1900), pp. 158—168.
4. ——. Summary in Botanische Zeitschrift, Abth. II, 1900.
5. ——. Ueber Levkojenbastarde. Botan. Centralbl. LXXXIV (1900), pp. 97—113.
6. ——. Ueber Bastarde zwischen Rassen von *Zea Mays*. Ber. d. deutsch. botan. Gesellsch. XIX (1900), pp. 211—220.
7. ——. Bibliotheca Botanica, No. 53 (1901).
8. Crampe. Zuchtversuche mit zahmen Wanderratten. An elaborate series of papers in Landwirthschaftliche Jahrbücher, VI (1877), XII (1883), XIII (1884), XIV (1885). The chief results quoted in the text are in the paper of 1877; but the whole series should be referred to.
9. Darwin, C. The Variation of Animals and Plants under Domestication (quoted from 2nd Edition, London, 1875).
10. Fischer, von. Iltis (*Mustela Putorius*) und Frett (*M. furo*). Der Zoologische Garten XIV (1873), pp. 108—112.
11. ——. Beobachtungen über Kreuzungen verschiedener Farbenspielarten innerhalb einer Species. Der Zoologische Garten XV (1874), pp. 361—374.
12. Gärtner, C. F. von. Versuche und Beobachtungen über die Bastarderzeugung im Pflanzenreich. Stuttgart, 1849.
13. Galton, F. The Average Contribution of each several Ancestor to the total Heritage of the Offspring. Roy. Soc. Proc. LXI (1897), pp. 401—413.
14. Giltay, E. Ueb. d. directen Einfluss d. Pollens auf Frucht- und Samenbildung. Jahrb. f. wiss. Botanik XXV (1893).

15. GOSS, J. Transactions Horticultural Society v (1848), pp. 234—236.

16. GUAITA, G. v. Versuche mit Kreuzungen etc. der Hausmaus. Ber. d. Naturf. Gesellsch. Freiburg x (1898).

17. HAACKE, W. Biologisches Centralbl. xvi (1896).

18. KNIGHT, T. A. Phil. Trans. 1799, pp. 195—204.

19. LAXTON, T. Observations on the Varieties effected by Crossing in the Colour and Character of the Seed of Peas. Report of Proceedings: International Horticultural Exhibition and Botanical Congress: London, 1866, p. 156.

20. ——. Journ. Roy. Hort. Soc. xii (1890), pp. 29—40.

21. MENDEL, G. Versuche über Pflanzen-Hybriden. Abhandl. d. Naturf. Vereins in Brünn iv (1865), pp. 3—47;
Also (edited by E. Tschermak) in Ostwald's Klassiker der exakten Wissenschaften No. 121;
Further, English translation by Bateson in Journal Royal Horticultural Society xxvi (1901).

22. NAUDIN, C. Nouvelles Recherches sur l'Hybridité dans les Végétaux. Nouvelles Archives du Muséum i (1865), pp. 25—176.

23. NOBBE, F., and others. Untersuchungen über den Einfluss der Kreuzbefruchtung auf die Nachkommenschaft. Die landwirthschaftliche Versuchsstationen xxxiv (1888).

24. PEARSON, K. The Law of Ancestral Inheritance. Roy. Soc. Proc. lxii (1898), pp. 386—412.

25. ——. On the Criterion that a given System of Deviations from the Probable in the case of a Correlated System of Variables is such that it can be reasonably supposed to have arisen from Random Sampling. Phil. Mag. Vol. l (1900), pp. 157—175.

26. RIMPAU, W. Kreuzungsprodukte landwirthschaftlicher Kulturpflanzen. Landwirthschaftliche Jahrbücher, Jahrg. xx (1891).

27. TSCHERMAK, E. Ueber künstliche Kreuzung bei *Pisum sativum*. Zeitschr. f. d. landwirthschaftliche Versuchswesen in Oesterreich, Jahrg. iii (1900), pp. 465—555.

28. ——. Weitere Beiträge über Verschiedenwerthigkeit der Merkmale bei Kreuzung von Erbsen und Bohnen. Zeitschr. f. d. landwirthsch. Versuchswesen, Jahrg. iv (1901), pp. 641—731.

29. ——. Ueber Züchtung neuer Getreiderassen, etc. Zeitschr. f. d. landwirthsch. Versuchswesen, Jahrg. iv (1901).

30. VRIES, H. DE. Sur la loi de disjonction des Hybrides. Comptes Rendus, cxxx (1900), pp. 845—847.

31. ——. Das Spaltungsgesetz der Bastarden. Ber. d. deutsch. botan. Gesellsch. xviii (1900), pp. 83—90.

32. ——. Ueber erbungleiche Kreuzungen. Ber. d. deutsch. botan. Gesellsch. xviii (1900), pp. 435—443.

EXPLANATION OF PLATES I. AND II.

PLATE I.

Figs. 1—6. Seeds of the hybrid Pea Telephone, the seed-coats removed, arranged so as to form a colour scale.

Figs. 7—12. Seeds of the hybrid Pea Stratagem, with the coat removed, forming a colour scale.

Figs. 13—18. Each figure shows a pair of cotyledons belonging to one seed of the Pea Telephone. The series shows the various degrees of difference in colour between the two cotyledons of the same seed.

Figs. 19—24. Peas in their seed-coats, showing the way in which the cotyledon colour is masked. 19—20, Telephone; 21, Telegraph; 22, Stratagem; 23, Pride of the Market; 24, Early Morn.

PLATE II.

PHOTOGRAPHS OF PEAS, SHOWING DEGREES IN THE DEVELOPMENT OF WRINKLES.

Figs. 1—4, degrees of wrinkling in Telegraph.

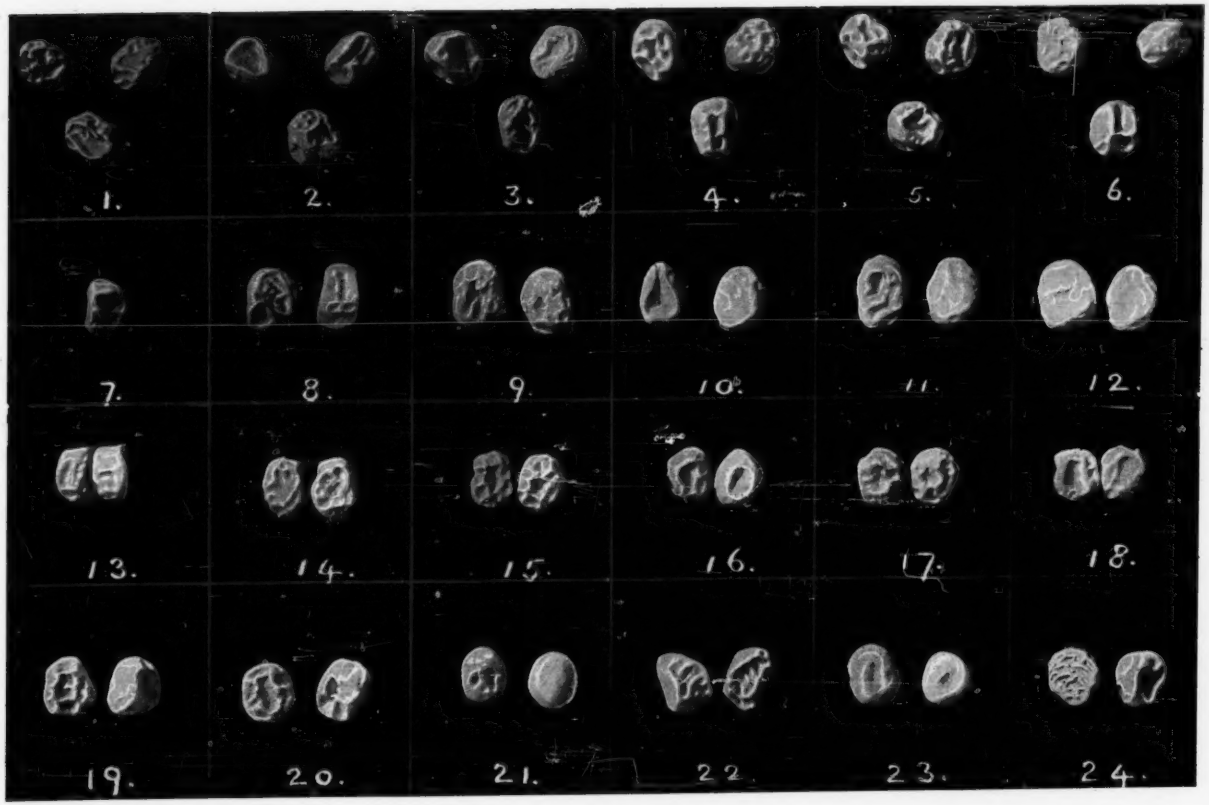
Figs. 5—6, seeds of Telephone.

Figs. 7—9, seeds of Stratagem.

Figs. 10—12, seeds of Daisy (=Stratagem × Giant Marrow).

Figs. 13—15, seeds of Lightning.

Figs. 16—18, seeds of Early Morn (=Daisy × Lightning).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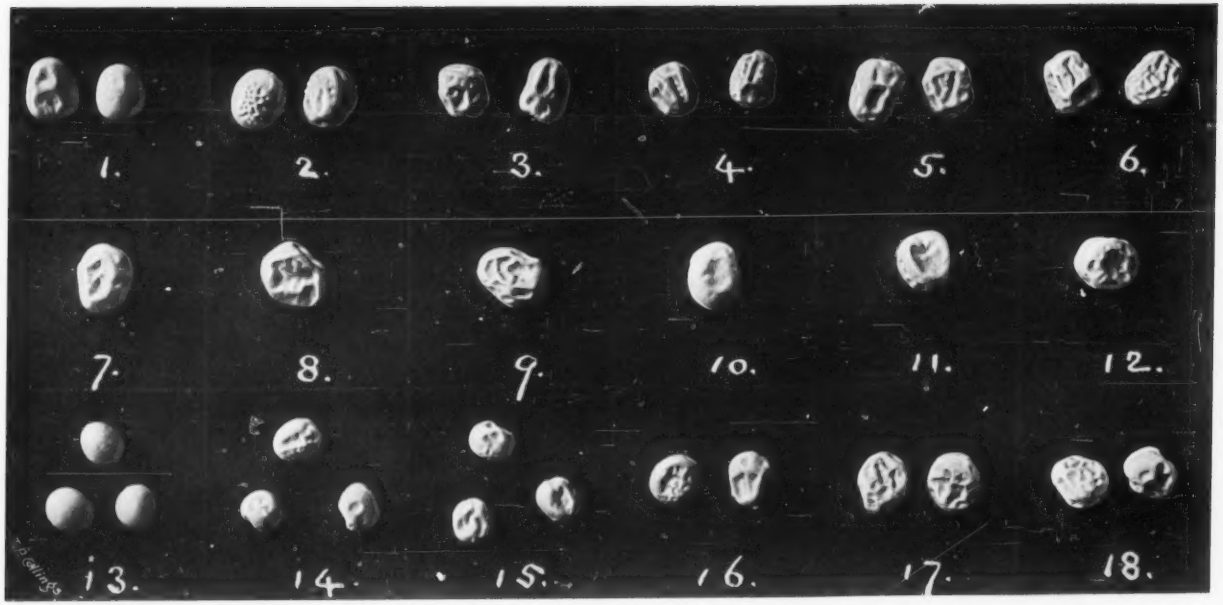
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MISCELLANEA.

I. Variability, Symmetry and Fertility in an abnormal Species.

In the jelly-fishes of the group Hydromedusæ four may be said to be the *normal* number of radii. There are ordinarily *four* radial canals, lips and gonads 90° apart. *Pseudoclytia pentata* (Mayer) differs from all other Hydromedusæ in that it normally has 5 radial canals, 5 lips and 5 gonads, 72° apart. The species occurs in great numbers at the Tortugas, Florida. We may speak of it as an "abnormal" species. In the *Science Bulletin of the Brooklyn Museum*, Vol. I. No. 1, Dr A. G. Mayer has given an interesting account of the variation of this species. The frequency of the different numbers of radial canals and of lips is given as follows:

Frequency.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Radial canals ...	—	1	8	56	860	64	6	1
Lips	2	5	18	123	798	49	1	—

From the correlation table the following results have been determined by Miss M. Lewenz of University College, London:

Organ	Mean	Standard Deviation	Correlation
Canals ...	5·004 ± ·009	·4413 ± ·0067	·3255 ± ·0191
Lips ...	4·868 ± ·012	·5559 ± ·0084	

The correlation between canals and lips is accordingly only moderate.

The frequency distribution of lips has been specially dealt with by Professor K. Pearson in a "Supplement to a memoir on Skew Variation," *Phil. Trans.* A, Vol. 197, p. 456. It forms an illustration of one of the curve-types omitted in his original memoir on that subject.

The individuals with 5 rays are nearly all radially symmetrical. The percentage of radially symmetrical individuals diminishes as we depart from this mode in either direction. In about two-thirds of the cases in which the canals have not a perfect *radial* symmetry they are nevertheless *bilaterally* symmetrical. There is a strong inherent tendency for some sort of symmetry.

Typically one gonad occurs on each radial canal, but one or more may fail to develop. Thus 730 medusæ with 5 canals had only 3080 gonads instead of the possible 3650.

The "coefficient of fertility" is $\frac{3050}{3610} = \cdot 8438$.

The coefficient of fertility differs according to the number of canals and to their regularity. Thus:

Number of canals	Coefficient of Symmetry	
	All individuals	Irregular individuals
3	·75	·555
4	·782	·69
5	·844	·63
6	·814	·69
7	·785	·57

It appears then that *the less typical an individual the less its fertility, and irregular individuals are more sterile than those having some sort of symmetry.* Thus the typical form and symmetry tend to be preserved.

It will be noted that this is a case of "reproductive selection," i.e. that there exists a correlation between fertility and the intensity of certain characters.

C. B. DAVENPORT.

II. Variation of the Egg of the Sparrow (*Passer domesticus*).

I HAD recently occasion for a special purpose to measure 180 clutches, containing 687 eggs of the house-sparrow. They were collected from Essex, Hertfordshire, Buckinghamshire, Berkshire, Surrey, Cambridgeshire, and even as far north as Nottinghamshire and Yorkshire. They were thus a fairly representative English series. The bulk of them were collected early in the year, and were undoubtedly first-nesting eggs. Having calculated out the constants for my series, it may not be without interest to compare them with constants obtained from the measurements of Professor Hermon C. Bumpus on both the American and English races of *Passer domesticus*. Professor Bumpus in 1896 measured 868 American and 868 English eggs and published* diagrams giving the frequency in length and index—breadth/length—of these series. He did not publish, as far as I am aware, the means and standard deviations of his frequency distributions, but concluded from his diagrams that the egg of the American sparrow is shorter and rounder than that of the English sparrow and further that it is more variable in length and shape. The greater variability of length and shape was judged not by the standard deviation (S. D.), or the coefficient of variability (C. of V.), but by the extent of the ranges. This method is, I believe, open to criticism†, and it seemed desirable to work out the numerical constants of Professor Bumpus' series. The results I have found are given in the table below, absolute lengths being in mms.

Sufficient data are not given for me to determine the S.D.'s of the breadths in Professor Bumpus' series, and as for the actual values of the breadths, I think, all we can say is that they must be very nearly equal and lie between 15·3 and 15·4. Now from these results we conclude:

(i) That the American egg is not more but *less* variable than the English in length.

* *Biological Lectures delivered at the Marine Biological Laboratory of Wood's Holl.* Summer Session, 1896. Boston, 1897.

† *The Chances of Death*, Vol. I. *Variation in Man and Woman*, p. 275.

(ii) That while the American egg is more variable in shape than Bumpus' English series, it is less variable than mine, although the difference between the two is really not significant.

(iii) That the American egg is sensibly shorter than the English egg, if judged by either English series.

(iv) That the American egg has probably slightly less breadth than the English egg.

(v) That having considerably less length and only slightly less breadth, the American egg appears rounder than the English egg.

	English Eggs		American Eggs
Character	Pearson (687)	Bumpus (868)*	Bumpus (868)
Mean Length ...	21·8229 ± ·0456	21·7368 ± ·0362	21·3065 ± ·0356
S. D. Length ...	1·1946 ± ·0322	1·0677 ± ·0256	1·0481 ± ·0252
C. of V. Length	5·47	4·91	4·92
Mean Breadth...	15·5052 ± ·0200	15·3683 ± [?]	15·3443 ± [?]
S. D. Breadth ...	·5245 ± ·0141	[?]	[?]
C. of V. Breadth	3·38	[?]	[?]
Index	71·2150 ± ·1433	70·8662 ± ·1092	72·1843 ± ·1250
S. D. Index	3·7560 ± ·1013	3·2155 ± ·0772	3·6831 ± ·0884

If we put aside Professor Bumpus' colour and shape appreciations†, I think we must conclude from the actual measurements that there is no evidence that the American egg is more variable than the English egg. On the contrary it appears slightly less variable, as we might expect if the sparrow imported into America had been largely drawn from one district of the mother country. In the next place the American egg does appear to be ·5 mm. shorter than the English. This might be due to (*a*) direct selection in America, or to (*b*) absence of the periodic selection we may suppose to maintain the stability of the species in England. But it may equally likely be due to original import into America of an English local race, to good or bad seasons in England or America in the year the eggs were collected, or to a large preponderance of second nestings in the clutches measured by Professor Bumpus. All these points could, I think, be investigated and answered. Professor Bumpus has opened a most interesting field of inquiry, and for the next few years arrangements ought to be made for the annual measurement in both America and England of good series of sparrows' eggs, with due record of district, probable nesting and climate of previous winter and spring. We might then learn the effect of each of these factors in both America and England, and at least put on record valuable results for comparison with similar measurements 20 or 30 years hence. Only in this way shall we be able to ascertain whether there are persistent sources of differentiation at work.

K. PEARSON.

* I could find only the indices of 867 English eggs given.

† The extraordinary variety of mottling and tint on my eggs was commented on by all who saw them; I consider that it varied a good deal with the district. I found no method of quantitatively measuring it. It disappeared to some extent with time and I daresay something of the colour would be liable to fade if the eggs were preserved for any time and transported.

III. Note on Variation in Leaves of Mulberry Trees.

ATTENTION has frequently been drawn to the difference in type of leaves growing on different parts of trees, and under the heading of *heterophylly* it is a topic familiar to botanists. It is only recently, however, that it has become a subject for statistical investigation. We may refer, for example, to the researches of De Brugher on "light" and "shade" leaves of the beech*, and to the measurements of A. Heyer on the needle of the pine embodied in Dr Ludwig's paper in the first number of *Biometrika*†. The present note deals with a similar case; it is hoped that it may serve as a preliminary study for a more extensive statistical investigation when further material is next year available. The importance of such investigations into a possible correlation between the character and situation of like organs is manifest if leaves are to be used as "homotypes" in the study of *homotyposis*.

The leaves of the common Mulberry appear to vary greatly according to their position on the tree, if I may judge from five trees which I have examined, and three of which information has been sent me.

Fig. I shows the usual, and greatly preponderating type of leaf, which on a large tree varies in size from about 3 to about 6 inches in length. In many there is a slight bulge or fulness at one place near the top, Fig. X, *A*, *B*, *C*: so that the leaf could not be pressed without some overlapping. Fig. III shows the greatest variation from the usual type that I have found: and Fig. II an intermediate form.

So far as I have observed, the deeply incised leaves (Fig. III) are found (*a*) largely on small twigs growing immediately from the main trunk and near its bottom, (*b*) occasionally on larger twigs or small boughs growing among the lower boughs: one has been found at the end of a low bough, but *never* high up on the tree.

The intermediate forms (Fig. II) are found (*a*) on small twigs along with No. III, (*b*) at the end of the lower boughs, and (*c*) apparently occasionally, but very rarely, at the end of higher boughs. The usual type is found in great preponderance over the whole tree: and the small twigs generally show this type too, though it is possible to find twigs with more or less incised leaves only.

With the assistance of Miss Eleanor Hobhouse the leaves of 8 mulberry trees of very different ages were examined. From these few trees it would appear that

(*a*) The age of the tree does not affect the matter: as the two youngest trees had very little variation, but the next in age had a very great deal.

(*b*) Well-grown upright trees with no ground or trunk shoots seem to have but little variation.

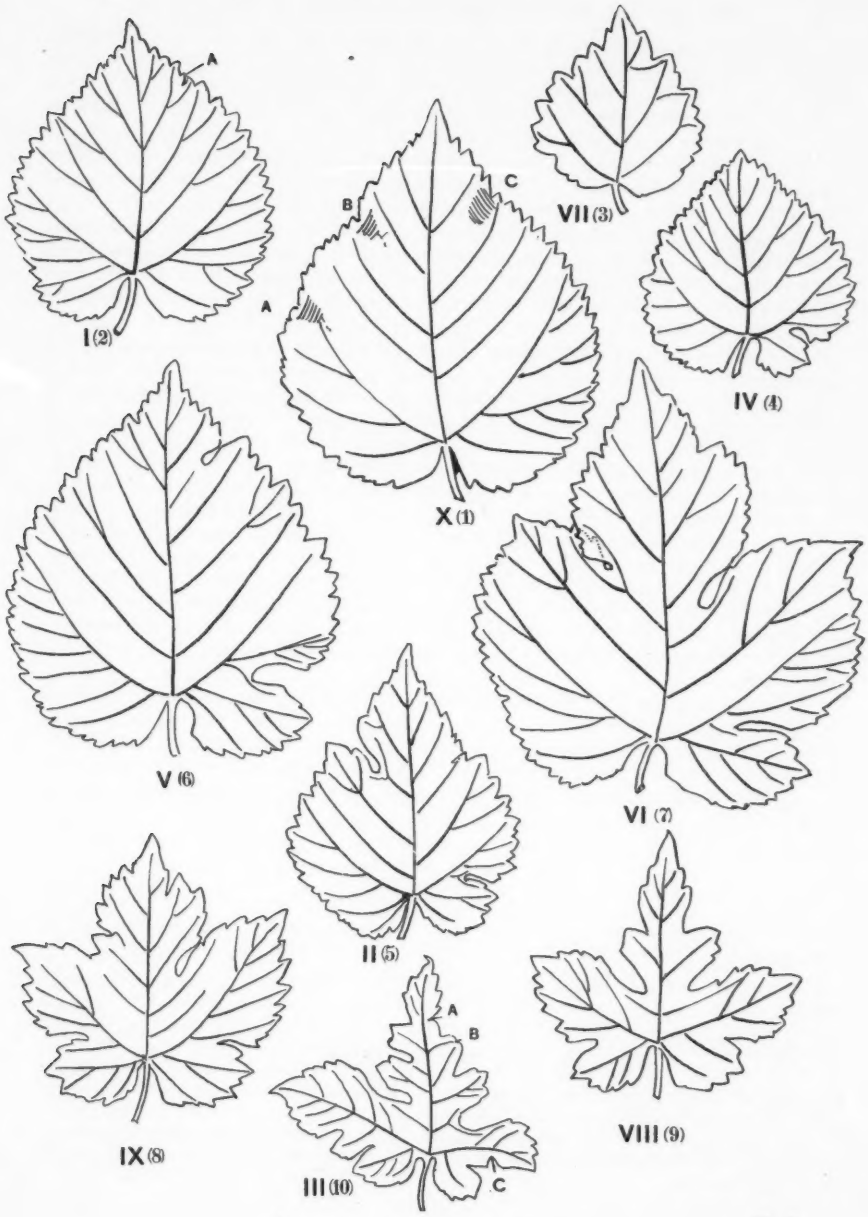
(*c*) Where the greatest number of cut leaves appears, the most deeply-cut leaves are also found.

(*d*) The lowest boughs and twigs have the greatest variation in respect of incision.

(*e*) The types melt into one another, and it would I think be easy to make a complete scale between Fig. X with its three bulges, and Fig. III with its emaciated form. In fact, the figures here given, if arranged according to the numbers in brackets, will form a very fairly continuous gradation from one extreme to the other; but I question whether any *one* of the trees examined would yield the complete scale.

AGNES FRY.

* *Bot. Jaarboek Dodonaea te Gent*, Vol. XI. p. 116, 1899.
† *Biometrika*, Vol. I. p. 20 *et seq.*


A
I (2)
B
C
A
X (1)
VII (3)
IV (4)
V (6)
VI (7)
II (5)
IX (8)
A
B
III (10)
C
VIII (9)

IV. On the Modal Value of an Organ or Character.

The mode has been defined as that value of an organ or character for which the frequency per unit variation of the organ or character is a maximum. The mode is so important for many purposes that its accurate determination is essential. Unfortunately rough and often practically worthless determinations of its value have been given somewhat frequently in recent papers.

It has to be remembered that in nine cases out of ten the mode cannot be settled by inspection of the observations, and that its calculation involves processes far more elaborate than those necessary to find the mean, and unless such processes are used the probable error of the mode will be so large as to render its determination of no practical service*.

The method frequently adopted is the following: The whole range of the organ measured is, say, 6 to 9 mm. This is divided into elementary ranges of ·4 or ·5 mm., and 500 to 1000 individuals being measured they are grouped into these elementary ranges. One of these elements exhibiting a maximum frequency, the mid-point of this element is selected as the modal value of the organ in question.

Now such a frequency grouping as that referred to is ample for a good determination of the mean value of the organ or, with the use of the proper corrective term, for that of the standard deviation. It is generally idle for determining the mode by inspection. The mode will extremely rarely be at the mid-point of the element of observed maximum frequency, and in very many cases will not lie in that element at all. To be reasonably certain that it does, there must be a very large preponderance of frequency in that element. In order to recognise the truth of the first statement, i.e. that the mode does not bisect the element of maximum frequency, consider the three consecutive frequencies 52, 93 and 84 which actually occur in a certain system of 400 measurements†. The group 93 falls into a certain range of 8·15 to 8·55 mm. and the modal value is supposed on inspection to fall into this range. If we accept this, it will surely fall nearer to the 84 than to the 52 side of the range, and therefore it is clearly erroneous to place it in the middle at 8·35. We might allow for this bias of the mode by interpolating a curve through the tops of ordinates of 52, 93 and 84 and finding its maximum ordinate, or better still by taking a curve, the *areas* of which on the corresponding ranges had these values.

This might give fairly good results if 52, 93 and 84 were the true values of the frequencies of the given ranges for the whole population. But they are not; they are subject to really considerable errors of random sampling. Let us find these errors. If an individual falls 93 times in 400 trials into a certain group, its chance of falling into this group is $\frac{93}{400}$, and the chance that it will not is $\frac{307}{400}$. Hence the S.D. of the random sampling of this group is $\sqrt{400 \times \frac{93}{400} \times \frac{307}{400}} = 8{\cdot}45$. Similarly the S.D.'s of the 84 and 52 groups are 8·15 and 6·73 respectively. Multiplying by ·67449 to obtain the probable errors our system of frequencies reads:

$$52 \pm 4{\cdot}54, \quad 93 \pm 5{\cdot}70, \quad 84 \pm 5{\cdot}49.$$

Now it will be clear that three numbers subject to probable errors of this kind can hardly give a very accurate determination of the mode! Indeed the difference between 93 and 84 is 9, and the probable error of this difference is 8·97‡. Thus the difference is sensibly equal to the

* The whole subject is well discussed by G. U. Yule: "Notes on the History of Pauperism," *Journal R. Stat. Soc.* Vol. LIX., pp. 318—357. See especially the "Supplementary Note on the Determination of the Mode," with illustration of various approximate determinations (p. 398), and also the account of the discussion on this point at the meeting (p. 356).

† Left marginal length of left-handed fiddler crabs, Diagram No. 44, of Robert M. Yerkes' paper: "A Study of Variation in the Fiddler Crab, *Gelasimus Pugilator*, Latr.," *Proc. American Academy of Arts and Sciences*, Vol. XXXVI. p. 429.

‡ It must be remembered that the errors in 93 and 84 are negatively *correlated*, and that accordingly the probable error of the difference is not the square root of the sum of the squares of the probable errors of 93 and 84.

probable error of the difference, or the odds are only about 3 to 1 against the 84 group actually coming out larger than the 93 group. In other words in every four random samples of this material we should find on the average the mode in one case (if judged by inspection) to fall in the 84 and not the 93 element! Thus large frequencies about the modal value are subject to large absolute probable errors, and unless we have investigated the probable error of the differences of such frequencies, we shall have no security for even having found the elementary range within which the mode really lies. In fact, if we want to find the mode satisfactorily we must take into account the frequency in elements lying outside the groups 52 and 84. Five groups will be better than three, seven than five, and so on. But where are we to stop? Clearly it will be best to take all the component frequencies, or *the mode can only be found with the maximum of accuracy when we deduce it like the mean from the whole series of observations.* It will rarely if ever lie at the mid-point of the group of *apparent* maximum frequency of observation, and very often will lie outside the range of this group altogether. It is quite fallacious to suppose it a constant of the distribution easily determined by inspection. To discover it involves some theory of the nature of the distribution of the frequency or some interpolation hypothesis; it cannot be found until the errors of random sampling have been smoothed by some such process.

For practical purposes the *median* is one of the easiest quantities to determine, and this can be found in a very few minutes from inspection of the measurements, i.e. count half the observations from either end of the frequency distribution, and this will land the counter part-way, say n individuals, into some elementary frequency group. Look out the individuals in the observation-book falling into this group and arrange them in order of size, the nth individual from the proper end of the group either gives the median value of the organ (total number of individuals odd) or we place the median value (total number of individuals even) mid-way between the n and $n+1$th individuals. If the median has been found as well as the mean, then a quite good value of the mode may be deduced by remembering that the median lies between the mode and the mean and that the distance from the median to the mode is double the distance from the mean to the median; this is close enough for practical purposes in the majority of frequency distributions*. Unless the mode be determined in this manner or from a complete treatment of the frequency distribution the mere tabulation of modes by inspection seems of small value, and the reasoning upon modes so determined liable to lead to fallacious conclusions.

K. PEARSON.

V. On the Change in Expectation of Life in Man during a period of circa 2000 years.

It is well known that the expectation of life at each age has changed in England very sensibly during the last 50 years—Farr's table differs very considerably from Ogle's table. The same remark applies, if we compare the Registrar-General's life table for 1881–90 with that of J. P., F.R.S., based on the London bills of mortality for 1728 to 1757†. But an opportunity has occurred for comparing the expectation of life in man at an interval of nearly 2000 years. The change that has taken place in this period cannot fail to be one of the greatest interest from the standpoint of evolution.

Professor Flinders Petrie has drawn my attention to the fact that the ages at death of a certain number of Egyptian mummies in the Roman period have been recorded and are published

* *Phil. Trans.*, Vol. 186, A. p. 375. Formulae for determining the mode and the probable error of its determination from the moments directly are given in a paper: "On the Mathematical Theory of Errors of Judgment, with Special Reference to the Personal Equation," in type for the *Phil. Trans.* See also *R. S. Proc.* Vol. 68, p. 369.

† See *R. S. Proc.* Vol. 67, p. 169.

by Professor W. Spiegelberg of Strassburg in his work: *Aegyptische und griechische Eigennamen aus Mumienetiketten*, 1901.

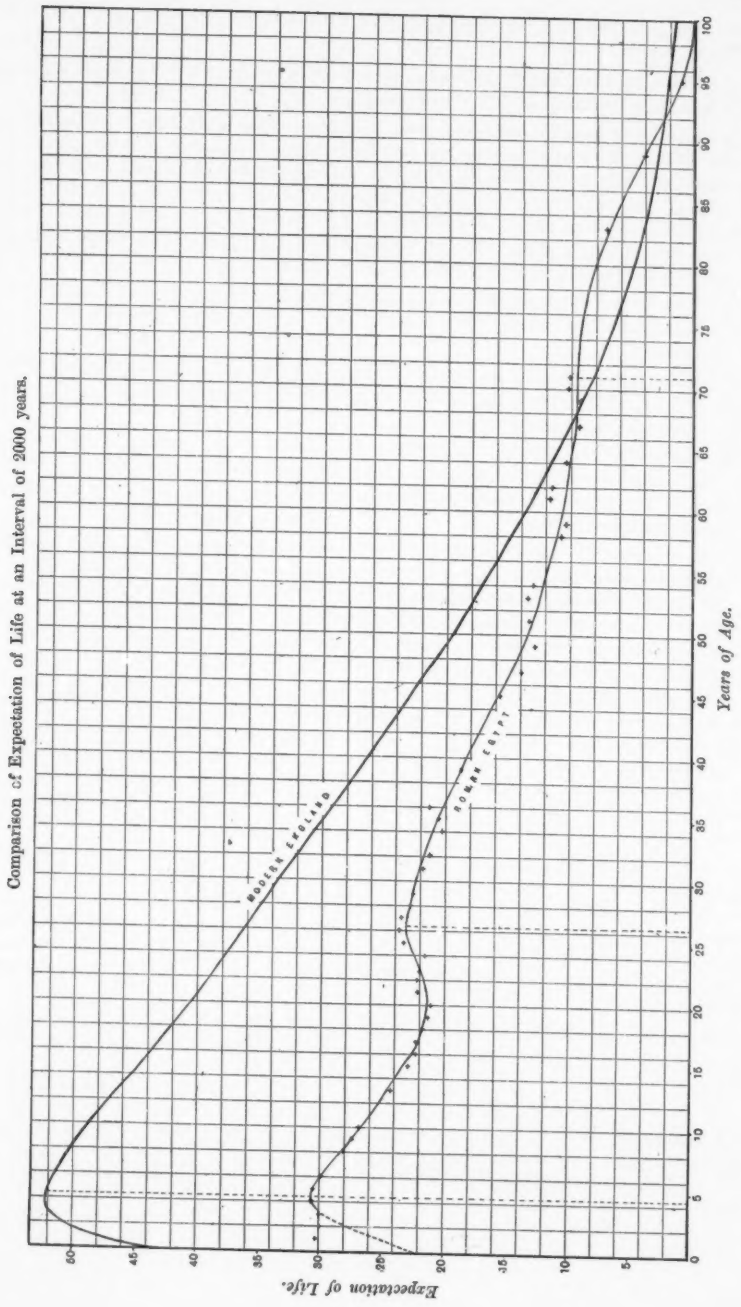
I presume that the individuals whose ages are thus preserved would not belong in bulk to the lowest nourished classes of the Egyptian population, so that they do not represent in any way a selected short-lived class. If anything we may suppose them to belong to the rather better classes, and at a time when a fairly settled civil government was in existence. We may reasonably treat the material as a fair sample of what one of the most stable populations of 1900 to 2000 years ago could manage in the matter of longevity. Unfortunately the material is very sparse. Professor Petrie provides me with the following table:

Age at Death	♂	♀	Age at Death	♂	♀
1½	1	1	33	2	1
1⅔	1	—	35	—	3
2	2	—	36	2	2
3	3	2	37	1	—
4	4	3	40	3	3
5	3	—	46	1	—
6	—	2	48	2	—
7	1	—	50	5	1
8	—	—	52	3	1
9	—	1	54	—	1
10	1	1	55	2	1
11	1	1	59	1	—
14	2	1	60	4	1
16	1	1	62	1	—
17	2	3	63	1	—
18	—	2	65	2	—
19	1	2	68	2	—
20	2	3	70	—	1
21	1	6	70·5	1	—
22	2	2	72	3	—
23	1	2	84	1	—
24	2	—	90	1	—
25	5	4	96	—	1
26	4	1			
27	1	1		82	59
28	—	1			
29	1	1			
30	1	2		Total	141
32	1	—			

In dealing with this table I have not ventured to separate the ♂ and ♀ mortality, the numbers are far too insignificant. In the second place I have used expectation of life, and not a mortality table, because at any rate after two years of age we get up to a good age the means of more or less considerable numbers. Evidently there are hardly any entries in the first year of life, there is no fair representation of infantile mortality. Baby mummies, if they ever existed, have disappeared out of all due proportion.

In the diagram I have indicated by a small cross each expectation of life that it was possible to calculate, and I have further placed upon the diagram the English expectation of life based upon the average of male and female*. Now if we neglect infancy and extreme old age we really get a very fair sweep showing the general trend of the Egyptian mortality. I have very roughly adjusted the data representing them by the continuous curve. No elaborate adjustment was

* Ogle's Table, 1871—80.


Comparison of Expectation of Life at an Interval of 2000 years.
MODERN ENGLAND
ROMAN EGYPT
Expectation of Life.
Years of Age.
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100

permissible, but from this adjustment, or even without it, the following results are clearly obvious :

(i) That up to the age of 68 the English expectation of life is greater, and in the earlier part of life immensely greater, than that of 2000 years ago.

In the course of those centuries man must have grown remarkably fitter to his environment, or else he must have fitted his environment immeasurably better to himself. No civilised community of to-day could show such a curve as the civilised Romano-Egyptians of 2000 years ago exhibit. We have here either a strong argument for the survival of the physically fitter man, or for the survival of the civilly fitter society. Either man is constitutionally fitter to survive to-day, or he is mentally fitter, i.e. better able to organise his civic surroundings. Both conclusions point perfectly definitely to an evolutionary progress.

(ii) The Egyptian curve differs fundamentally from the English in exhibiting apparently three maxima instead of one. These maxima must arise from the mortality curve itself being multinodal.

The first maximum is at about 4·5 years of age and corresponds to the English at about 4 years of age*. This is associated with the point where the especial dangers of infantile and childhood mortality have been surpassed.

The second Egyptian maximum occurs between 26 and 27 and possibly marks the period where the dangers of youthful mortality have been surmounted. The mortality of youth in our English case is a rather small component (greater for the French) and is centred at 22·5 years, becoming insignificant at 35.

I take it that this was much emphasised in Ancient Egypt, and reached its maximum considerably earlier. The third Egyptian maximum is less definite, because there are so few cases of extreme longevity to base the means upon. But it seems to me, looking at the points between 60 and 70, clearly impossible for the expectation of life curve to approach the axis without a point of inflexion and a maximum somewhere about 71—possibly earlier, even as early as 65, but the data are not sufficient to determine the point at all closely†. If this third maximum really exists, it would probably mean that the "mortality of middle life" which in the case of England is centred about 41·5 and ceases about 70 was much more definitely marked in Egypt. [See my paper in the *Phil. Trans.*, Vol. 186, p. 408, and especially Plate 16.] That the expectation of life for a Romano-Egyptian over 68 was greater than for a modern English man or woman is what we might expect, for with the mortality of youth and of middle age enormously emphasised only the very strongest would survive to this age. Out of 100 English alive at 10 years of age 39 survive to be 68; out of 100 Romano-Egyptians not 9 survived.

Looking at these two curves, we realise at a glance either the great physical progress of man, which enables him far more effectually to withstand a hostile environment, or the great social and sanitary progress he has made which enables him to modify that environment. In either case we can definitely assert that 2000 years has made him a much "fitter" being. In this comparison it must be remembered that we are not placing a civilised race against a barbaric tribe, but comparing a modern civilisation with one of the highest types of ancient civilisation.

That a man of 25 years to-day lives on an average 15 more years than a man of 25 years did 2000 years ago is surely very strong evidence that man has progressed substantially in this period.

K. PEARSON.

* I have turned down the Egyptian curve in a perfectly arbitrary manner by a broken line to indicate to the reader, looking at the diagram, that the infantile mortality is practically unrecorded.

† I am inclined to think the Egyptian curve for a considerable period must be concave to the horizontal axis, instead of throughout convex as in the case of the English.